Heat Transfer: Convection

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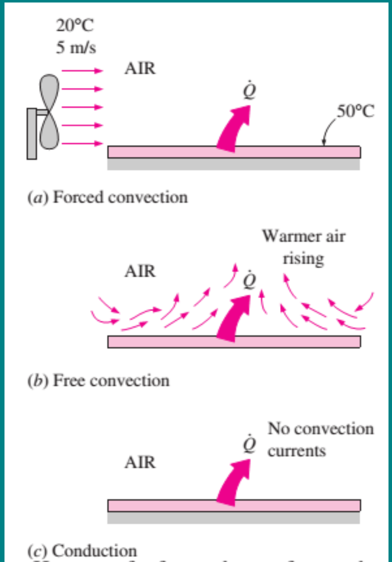
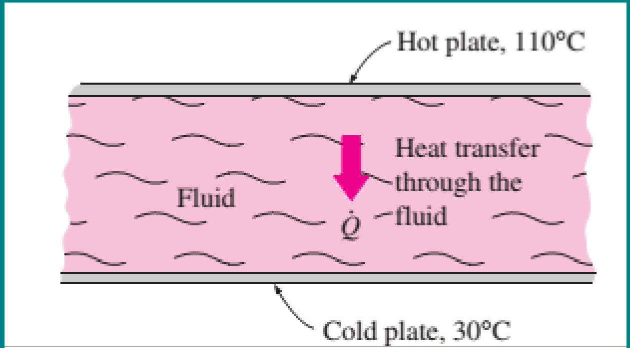
INTRODUCTION

Convection: Heat transfer **through a fluid** as a **medium** in the **presence of bulk** fluid motion



Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

Source: Cengel et al. 2010

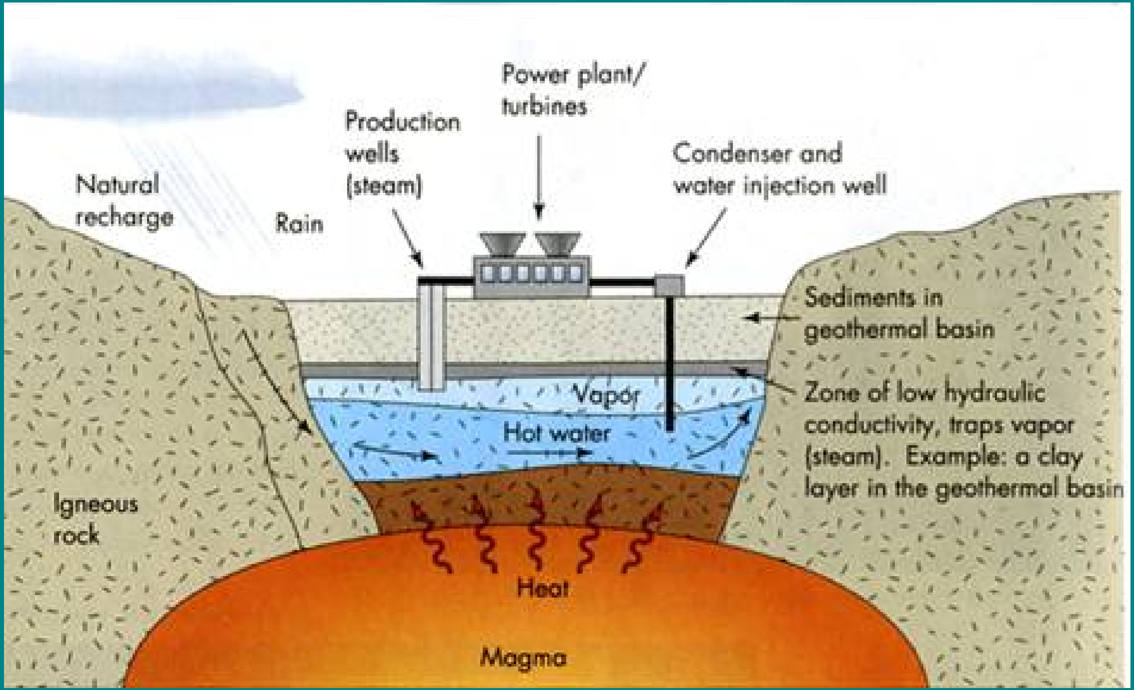


Heat transfer through a fluid sandwiched between two parallel plates.

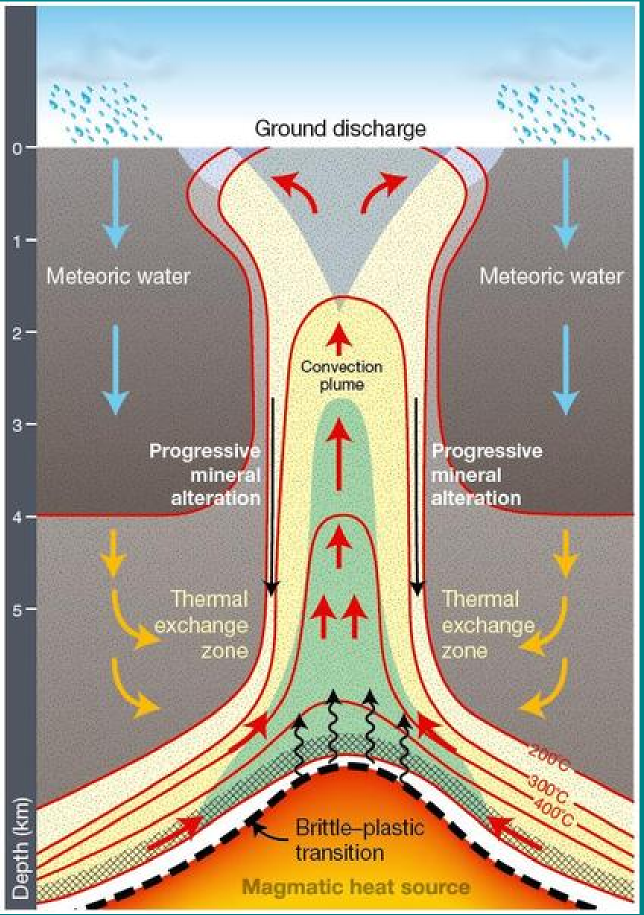
Source: Cengel et al. 2010

INTRODUCTION

Natural hydrothermal-convection systems



Source: Environmental Geology, Prentice Hall, 2000.



Source: www.gns.cri.nz

Elements of a geothermal system

- **Heat source**: Underlying the system at depth is hot magma.
- **Rising water**: Groundwater near the magma becomes heated and more buoyant than the surrounding colder waters and rises through pathways that lead to the surface (hot water is also less viscous than cold water).
- **Hot water plume**: The rising water is discharged at the surface through hot springs and steaming ground.
- **Interaction**: The rising fluids interact chemically with the surrounding rocks and their temperature is moderated by mixing with cooler water and by local boiling.
- **Counter flow**: The upward flow of hot water from depth creates a downward counter-flow in the surrounding area so the surrounding cold water moves downwards.
- **Convection system**: The movement of the two types of water create a circulating convection system that is a very efficient way to transfer heat.

RATE OF CONVECTION

The rate of convection heat transfer is expressed by Newton's law of cooling

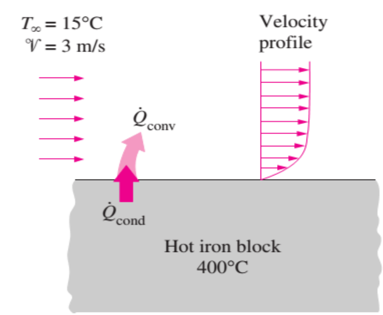
$$\dot{q}_{conv} = h(T_s - T_\infty) \quad \text{or} \quad \dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

where: $h =$ convection heat transfer coefficient, $W/m^2.$℃

$A_s =$ heat transfer surface area, m^2

$T_s =$ temperature of the surface, ℃

$T_\infty =$ temperature of the fluid sufficiently far from the surface, ℃



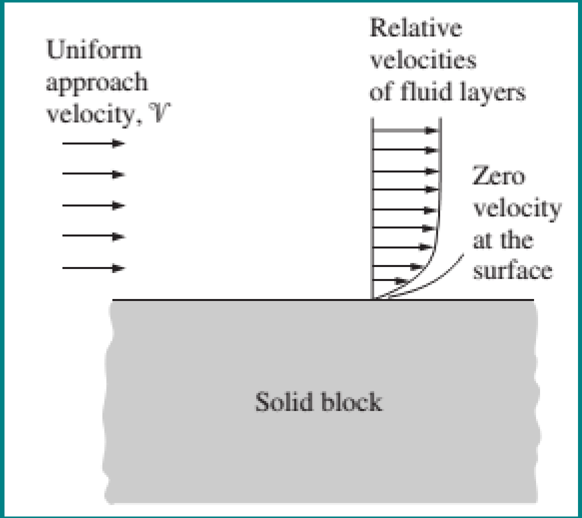
The cooling of a hot block by forced convection.

Source: Cengel et al. 2010

The cooling of a hot iron block with a fan blowing air over its top surface, we know that heat will be transferred from the hot block to the surrounding cooler air, and the block will eventually cool. We also know that the block will cool faster if the fan is switched to a higher speed. Replacing air by water will enhance the convection heat transfer even more.

PHYSICAL MECHANISM OF CONVECTION

- No-slip condition



Source: Cengel et al. 2010

Implication of the no-slip and the no-temperature jump conditions can be expressed as

$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid}\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

- No-temperature-jump condition

Nusselt Number (nondimensionalize the governing equations)

$$Nu = \frac{hL_c}{k}$$

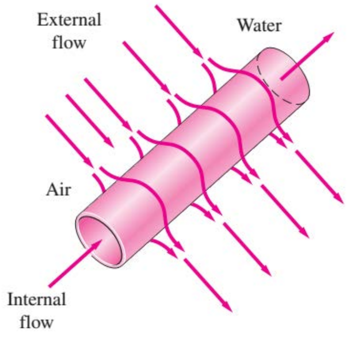
The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The larger the Nusselt number, the more effective the convection.

CLASSIFICATION OF FLUID FLOWS

1. Viscous versus Inviscid Flow

- Viscous: the effects of viscosity are significant
- Inviscid: flows of zero-viscosity fluids (frictionless)

2. Internal versus External Flow



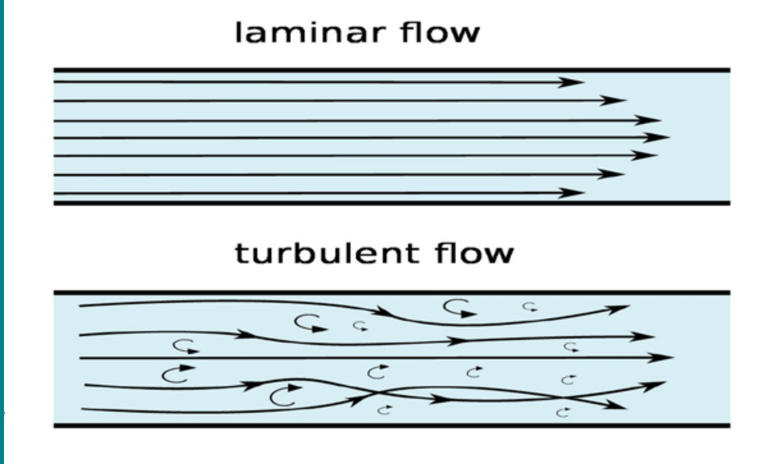
Internal flow of water in a pipe and the external flow of air over the same pipe.

Source: Cengel et al. 2010

3. Compressible versus Incompressible Flow

- Compressible: ρ varies
- Incompressible: ρ constant

4. Laminar versus Turbulent Flow



Source: www.cfdsupport.com

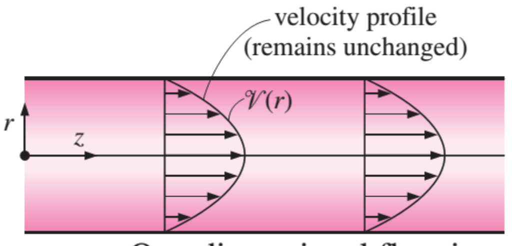
5. Natural (or Unforced) versus Forced Flow

- Natural: fluid motion is due to a natural means such as the buoyancy effect
- fluid is forced to flow over a surface or in a pipe by external means (such as pump, fans, etc)

6. Steady versus Unsteady (Transient) Flow

- Steady: no change with time
- Unsteady (Transient): the flow velocity and pressure are changing with time

7. One-, Two-, and Three-Dimensional Flows



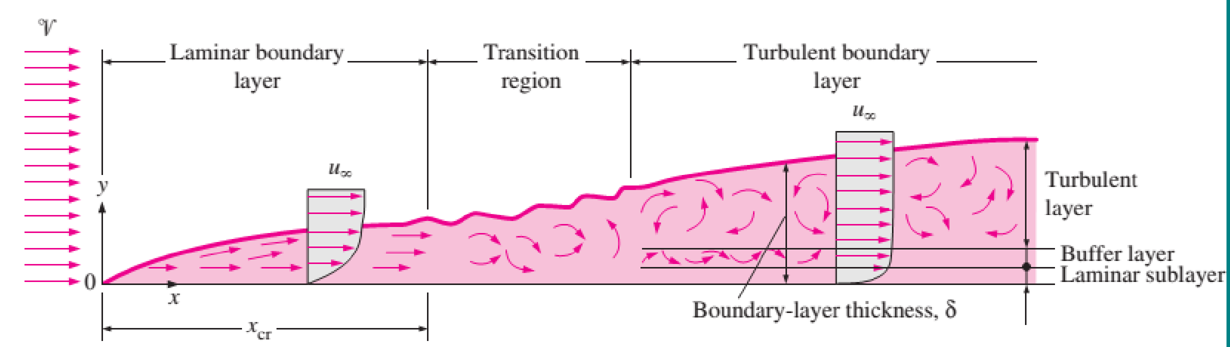
One-dimensional flow in a circular pipe.

Source: Cengel et al. 2010

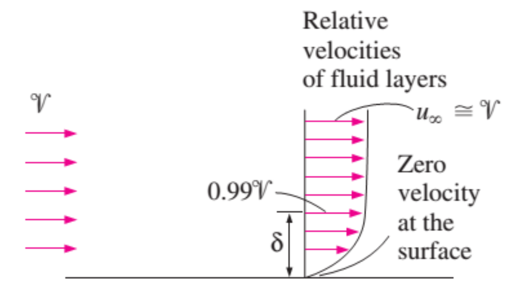
Fluid flow in a circular pipe is one-dimensional since the velocity varies in the radial r direction but not in the angular θ- or axial z-directions

VELOCITY BOUNDARY LAYER

The parallel flow of a fluid over a flat plate:



The development of the boundary layer for flow over a flat plate, and the different flow regimes.



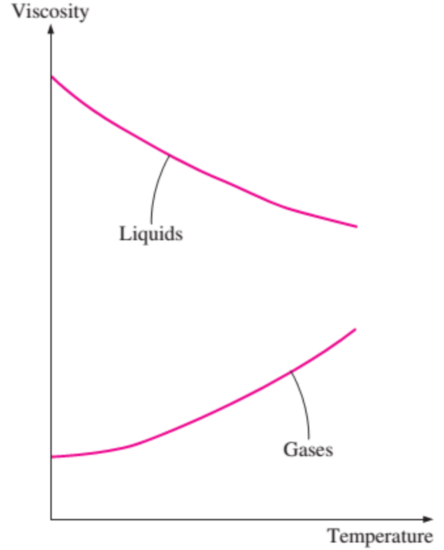
The development of a boundary layer on a surface is due to the no-slip condition.

Source: Cengel et al. 2010

Source: Cengel et al. 2010

The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into two regions: the **boundary layer region**, in which the viscous effects and the velocity changes are significant, and the **inviscid flow region**, in which the frictional effects are negligible and the velocity remains essentially constant.

Surface Shear Stress



The viscosity of liquids decreases and the viscosity of gases increases with temperature.

Source: Cengel et al. 2010

Friction force per unit area is called shear stress, and is denoted by τ.

- The practical approach in external flow is to relate τ_s to the upstream velocity V is:

$$\tau_s = C_f \frac{\rho V^2}{2} \quad (N/m^2)$$

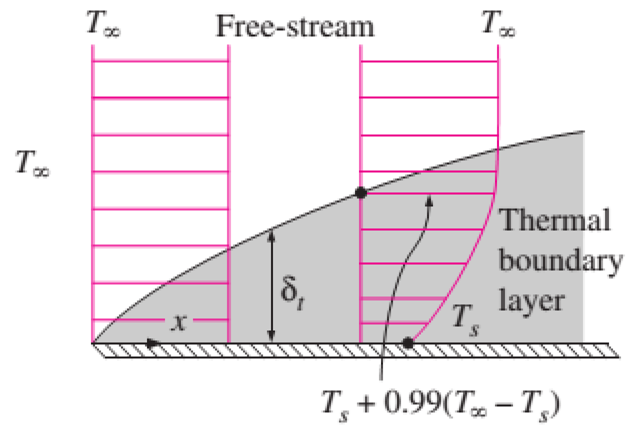
- The friction force over the entire surface:

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad (N/m^2)$$

C_f is the dimensionless friction coefficient (determined experimentally)

THERMAL BOUNDARY LAYER

<u>The parallel flow of a fluid over a flat plate:</u>



Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

Source: Cengel et al. 2010

The thickness of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference $T - T_s$ equals 0.99($T_\infty - T_s$).* Note that for the special case of $T_s = 0$, we have $T = 0.99T_s$ at the outer edge of the thermal boundary layer.

Prandtl Number (dimensionless parameter)

$$Pr = \frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ heat} = \frac{v}{\alpha} = \frac{\mu C_p}{k}$$

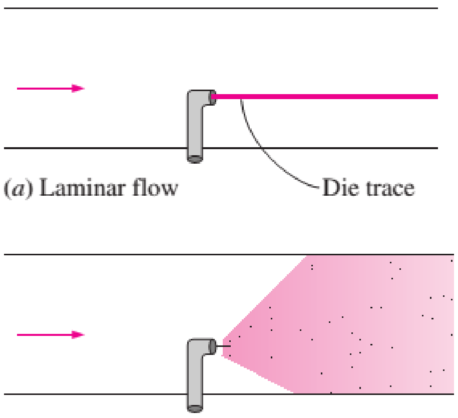
The Prandtl Number number represents the relative thickness of the velocity and the thermal boundary layers.

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

Source: Cengel et al. 2010

LAMINAR AND TURBULENT FLOWS



The behavior of colored fluid injected into the flow in laminar and turbulent flows in a tube.

Source: Cengel et al. 2010

- Laminar is characterized by smooth streamlines and highly-ordered motion
- Turbulent is characterized by velocity fluctuations and highly-disordered motion
- The transitionfrom laminar to turbulent flow does not occur suddenly. It occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent

Reynolds Number (dimensionless parameter)

$$Re = \frac{Inertia\ forces}{Viscous} = \frac{VL_c}{v} = \frac{\rho VL_c}{\mu}\ \text{(for external flow)}$$

- At large Reynolds numbers, the viscous forces cannot prevent the random and rapid fluctuations of the fluid (Turbulent)
- At small Reynolds numbers, the viscous forces are large enough to overcome the inertia forces and to keep the fluid "in line" (Laminar)
- **Critical Reynolds number,** at which the flow becomes turbulent

$$Re_{cr} = \frac{Vx_{cr}}{v} = \frac{u_\infty x_{cr}}{v} = 5\text{x}10^5 \quad \text{(flow over a flat plate)}$$

EXTERNAL FORCED CONVECTION

Drag and Heat Transfer in External Flow

- Friction and Pressure Drag

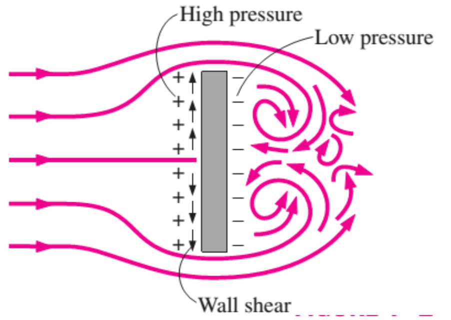
Drag is the force a flowing fluid exerts on a body in the flow direction

Lift is the components of the pressure and wall shear forces in the normal direction to flow tend to move the body in that direction, and their sum is called lift

$$Drag\ Coefficient: \quad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = C_{D,friction} + C_{D,pressure}$$

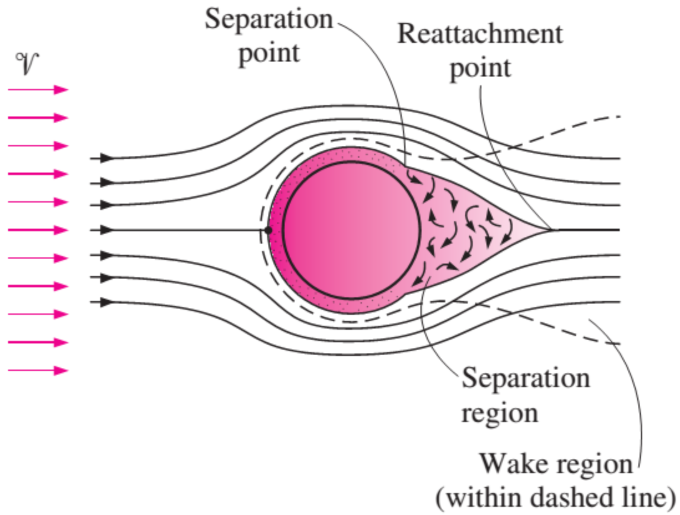
Friction drag: the part of drag that is due directly to wall shear stress τ_w

Pressure drag: the part of drag that is due directly to pressure P



Drag force acting on a flat plate normal to flow depends on the pressure only and is independent of the wall shear, which acts normal to flow.

Source: Cengel et al. 2010



Separation and reattachment during flow over a cylinder, and the wake region.

Source: Cengel et al. 2010

Flow over a cylinder:

- The low-pressure region behind the body where recirculating and back flows occur is called the **separation region**
- The region of flowtrailing the body where the effect of the body on velocity is felt is called **the wake region**

- Heat Transfer

Film temperature is the arithmetic average of the surface and the free-stream temperatures

$$T_f = \frac{T_s - T_\infty}{2}$$

When the average drag and convection coefficients are available, the rate of heat transfer to or from an isothermal surface can be determined from:

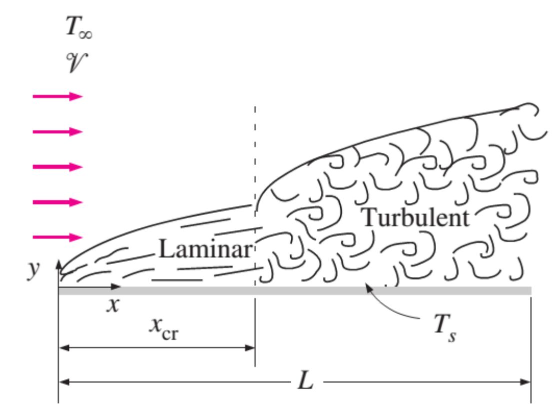
$$\dot{Q} = hA_s(T_s - T_\infty)$$

PARALLEL FLOW OVER FLAT PLATES

The transition from laminar to turbulent flow depends on *the surface geometry, surface roughness, upstream velocity, surface temperature, and the type of fluid*, among other things, and is best characterized by the Reynolds number.

$$Re_x = \frac{\rho V x}{\mu} = \frac{V x}{v}$$

For flow over a flat plate, the critical Reynolds number is $Re_{cr} = 5\text{x}10^5$



Laminar and turbulent regions of the boundary layer during flow over a flat plate.

Friction Coefficient

The average friction coefficient over the entire plate is:

Laminar $C_f = \dfrac{1.328}{Re_L^{1/2}}$ $Re_L < 5\text{x}10^5$

Turbulen $C_f = \dfrac{0.074}{Re_L^{1/5}}$ $5\text{x}10^5 \leq Re_L \leq 10^7$

The average friction coefficient for turbulent flow on rough surfaces $Re > 10^6$ (from experimental data by Schlichting):

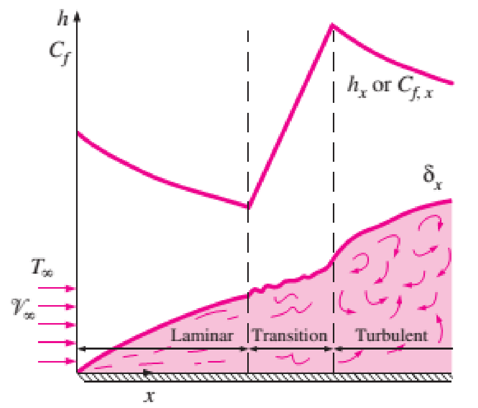
Rough surface, turbulent: $C_f = \left(1.89 - 1.62\, log\, \dfrac{\varepsilon}{L}\right)^{-2.5}$

Relative roughness, ε/L	Friction coefficient C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

*Smooth surface for Re = 10^7. Others calculated from Eq. 7-18.

For turbulent flow, surface roughness may cause the friction coefficient to increase severalfold.

Heat Transfer Coefficient



The variation of the local friction and heat transfer coefficients for flow over a flat plate.

The average Nusselt number over the entire plate

Laminar ($Re_L < 5\text{x}10^5$) : $$Nu = \frac{hL}{k} = 0.664 Re_L^{0.5} Pr^{1/3}$$

Turbulen ($5\text{x}10^5 \leq Re_L \leq 10^7$: $$Nu = \frac{hL}{k} = 0.037 Re_L^{0.8} Pr^{1/3}$$

$0.6 \leq Re_L \leq 60$)

A single correlation that applies to *all fluids* by curve-fitting existing data (proposed by Churchill and Ozoe) which is applicable for all Prandtl numbers:

$$Nu_x \frac{h_x x}{k} = \frac{0.3387 Pr^{1/3} Re_x^{1/2}}{\left[1 + \left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}}$$

Flat Plate with Unheated Starting Length

The local Nusselt numbers for both laminar and turbulent flows (using integral solution methods, by Kays and Crawford 1994) are determined to be:

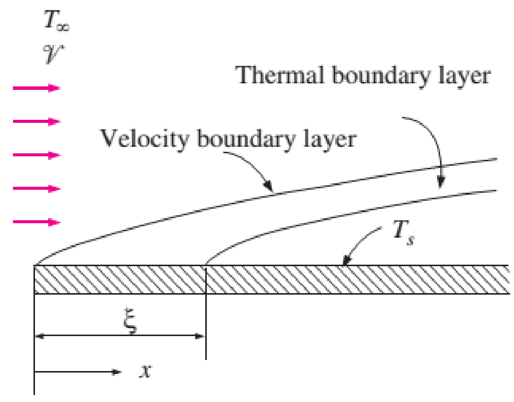
Laminar:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

Turbulent:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

The average convection coefficients (using numerical integration, by Thomas 1977) :

Laminar:
$$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent:
$$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L}$$



Flow over a flat plate with an unheated starting length.

Uniform Heat Flux

When a flat plate is subjected to uniform heat fluxinstead of uniform temperature, the local Nusselt number is given by

Laminar $Nu_x = 0.453\, Re_x^{0.5} Pr^{1/3}$

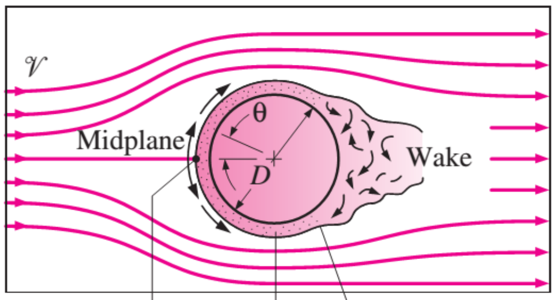
Turbulen $Nu = 0.0308 Re_x^{0.8} Pr^{1/3}$

When heat flux $\dot{q}_s$sis prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance xare determined from

$\dot{Q} = \dot{q}_s A_s$ and $\dot{q}_s = h_x[T_s(x) - T_\infty]$

FLOW ACROSS CYLINDERS AND SPHERES

Flow across cylinders and spheres is frequently encountered in practice, for example, the tubes in a shell-and-tube heat exchanger.



Typical flow patterns in cross flow over a cylinder.

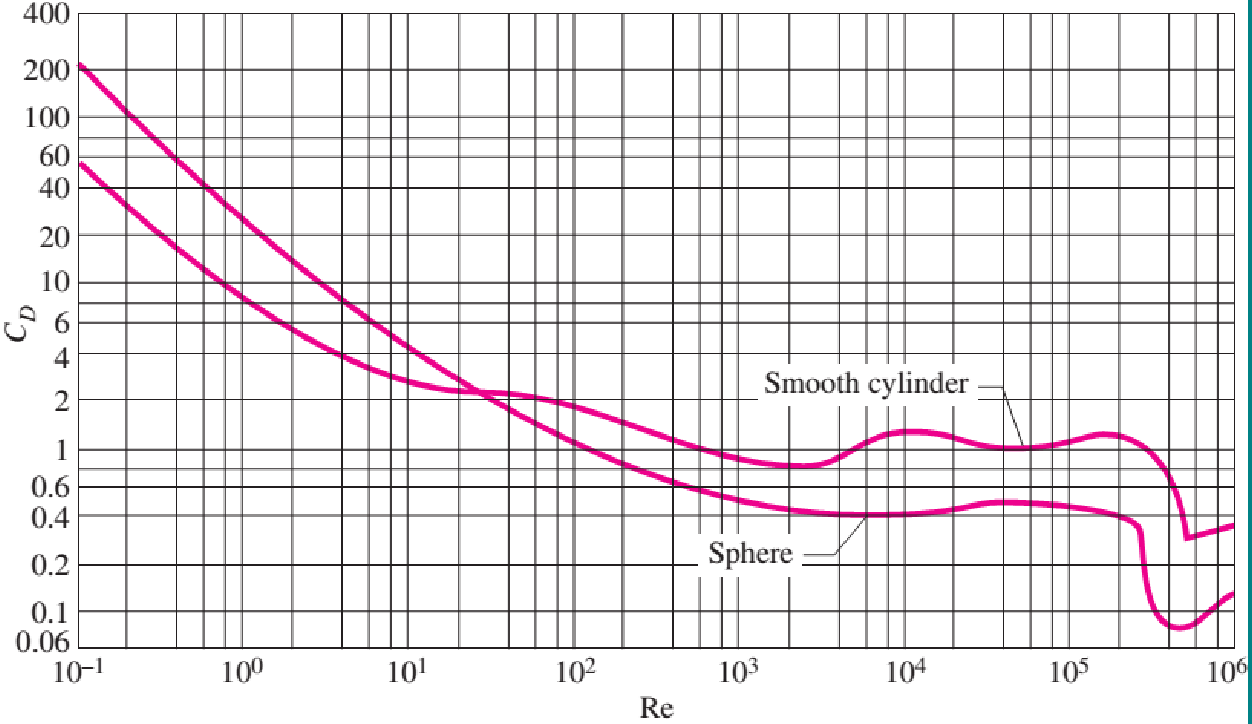
Reynolds Number

$$Re = \frac{VD}{\nu}$$

The critical Reynolds number for flow across a circular cylinder or sphere is $Re_{cr} \approx 2\text{x}10^5$

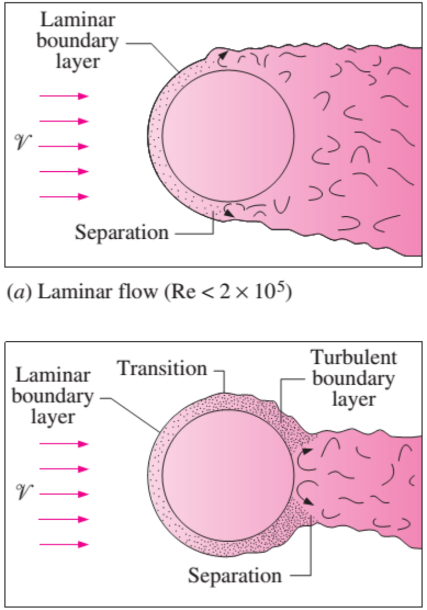
- Laminar $Re_{cr} \leq 2\text{x}10^5$
- Turbulent $Re_{cr} \geq 2\text{x}10^5$

The average drag coefficients C_D



Average drag coefficient for cross flow over a smooth circular cylinder and a smooth sphere (from Schlichting, Ref. 10).

Flow Separation



(*a*) Laminar flow ($Re < 2 \times 10^5$)

(*b*) Turbulence occurs ($Re > 2 \times 10^5$)

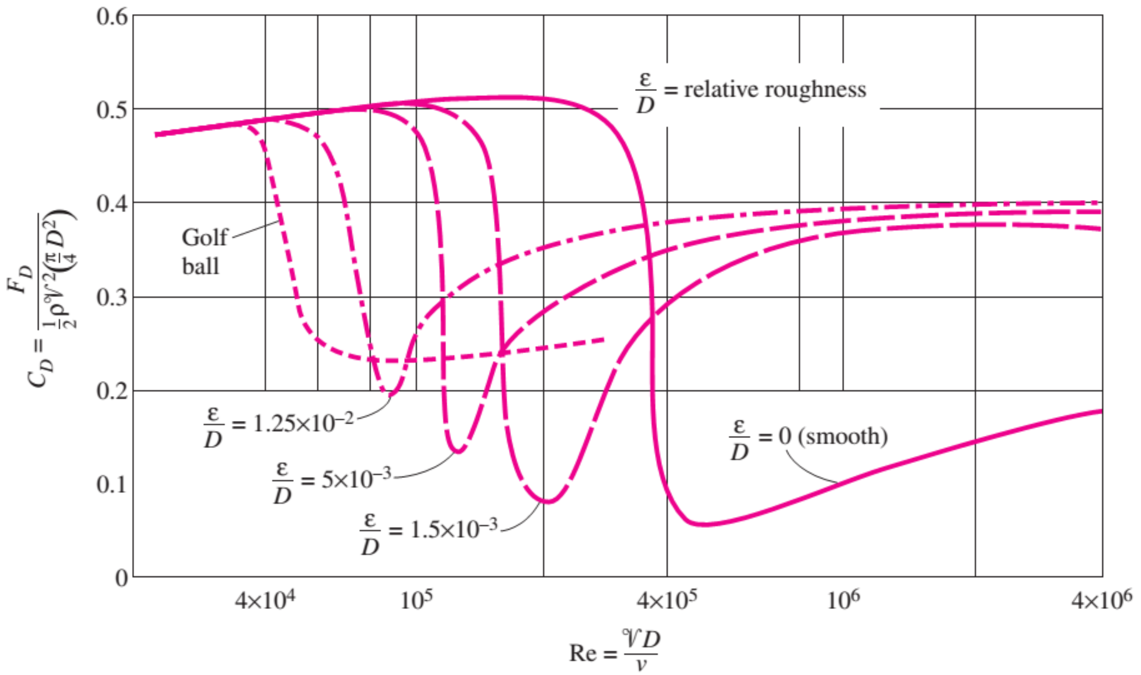
Turbulence delays flow separation.

Flow separation occurs at about (measured from the stagnation point)

- $\theta \approx 80°$ for laminar
- $\theta \approx 140°$ for turbulent

The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel further along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag

Effect of Surface Roughness



The effect of surface roughness on the drag coefficient of a sphere (from Blevins, Ref. 1).

HEAT TRANSFER COEFFICIENT

Nusselt number for cross flow over a cylinder (proposed by Churchill and Bernstein)

$$\mathrm{Nu}_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

For flow over a sphere, Whitaker recommends the following comprehensive correlation:

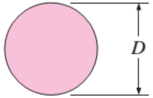
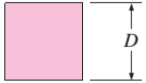
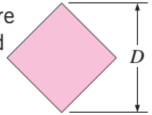
$$\mathrm{Nu}_{sph} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

The average Nusselt number for flow across cylinders can be expressed compactly as

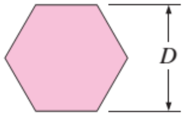
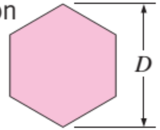
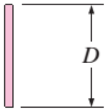
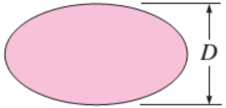
$$Nu_{cyl} = \frac{hD}{k} = CRe^{m}Pr^{n}$$

where $n = \frac{1}{3}$, the experimentally determined constants C and m are given from

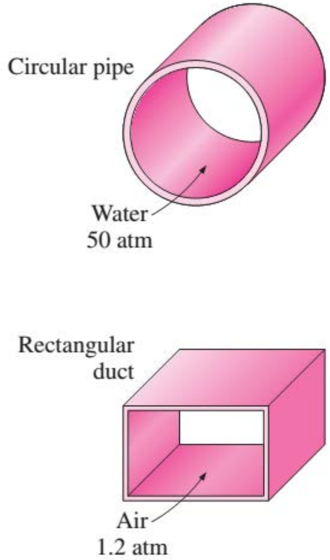
Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}Pr^{1/3}$ $Nu = 0.911Re^{0.385}Pr^{1/3}$ $Nu = 0.683Re^{0.466}Pr^{1/3}$ $Nu = 0.193Re^{0.618}Pr^{1/3}$ $Nu = 0.027Re^{0.805}Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}Pr^{1/3}$

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Hexagon 	Gas	5000–100,000	$Nu = 0.153 Re^{0.638} Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160 Re^{0.638} Pr^{1/3}$ $Nu = 0.0385 Re^{0.782} Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228 Re^{0.731} Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248 Re^{0.612} Pr^{1/3}$

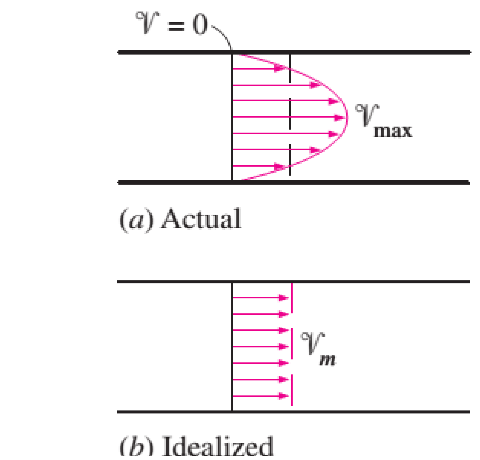
INTERNAL FORCED CONVECTION



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot.

- The circular tube gives the most heat transfer for the least pressure drop, which explains the overwhelming popularity of circular tubes in heat transfer equipment.
- Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small and the manufacturing and installation costs arelower

MEAN VELOCITY AND MEAN TEMPERATURE



Actual and idealized velocity profiles for flow in a tube (the mass flow rate of the fluid is the same for both cases).

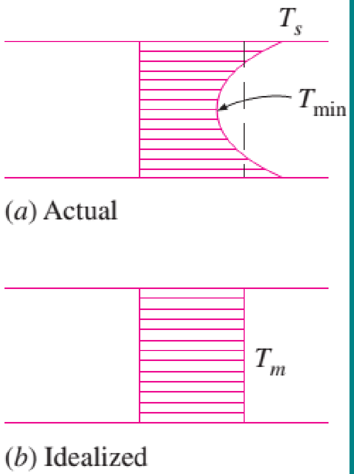
The fluid velocity in a tube changes from zero at the surface because of the **no-slip condition**, to a maximum at the tube center.

Average or **mean velocity** V_m remains constant for incompressible flow when the cross sectional area of the tube is constant.

For the flow through circular tube of radius R:

$$V_m = \frac{2}{R^2}\int_0^R V(r,x)r\,dr$$

MEAN VELOCITY AND MEAN TEMPERATURE



Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

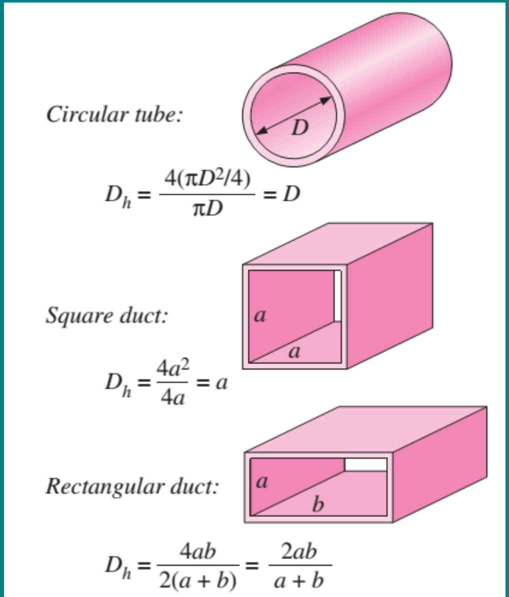
When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center

Average or **mean temperature** T_m will change in the flow direction whenever the fluid is heated or cooled.

For the flow through circular tube of radius R (density and specific heat of fluid are constant) :

$$T_m = \frac{2}{V_m R^2} \int_0^R T(r,x) V(r,x) r \, dr$$

Reynold Number



The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

For flow in a circular tube, the Reynolds number is defined as:

$$Re = \frac{\rho V_m D}{\mu} = \frac{V_m D}{v}$$

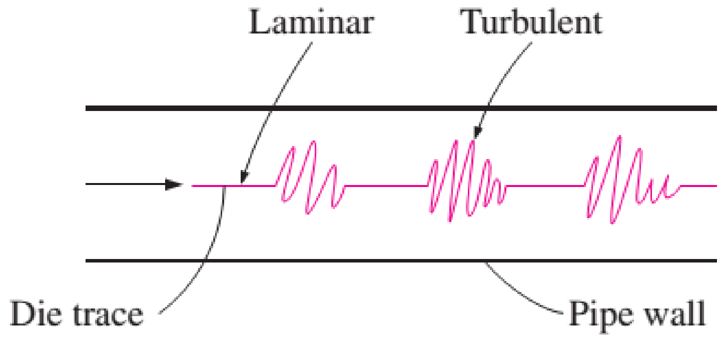
For flow in noncircular tubes, Reynold and Nusselt are based on **hydraulic diameter** D_h

$$D_h = \frac{4A_C}{p}$$

A_C is the cross sectional area of the tube

p is the perimeter.

Laminar and Turbulent Flow In Tubes

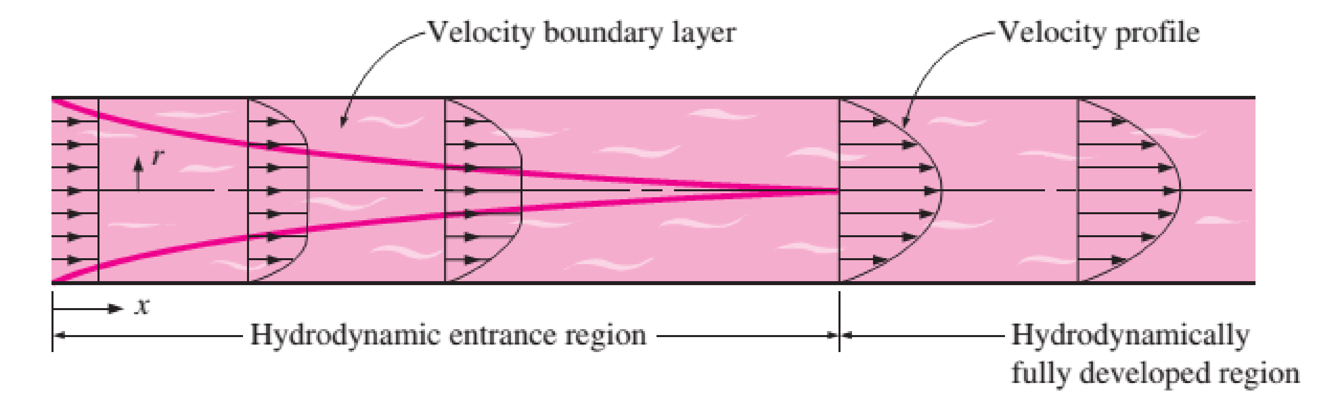


In the transitional flow region of $2300 \leq \mathrm{Re} \leq 4000$, the flow switches between laminar and turbulent randomly.

The transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by surface roughness, pipe vibrations, and the fluctuations in the flow.

$Re < 2300$	laminar flow
$2300 \leq Re \leq 10000$	transitional flow
$Re > 10000$	turbulent flow

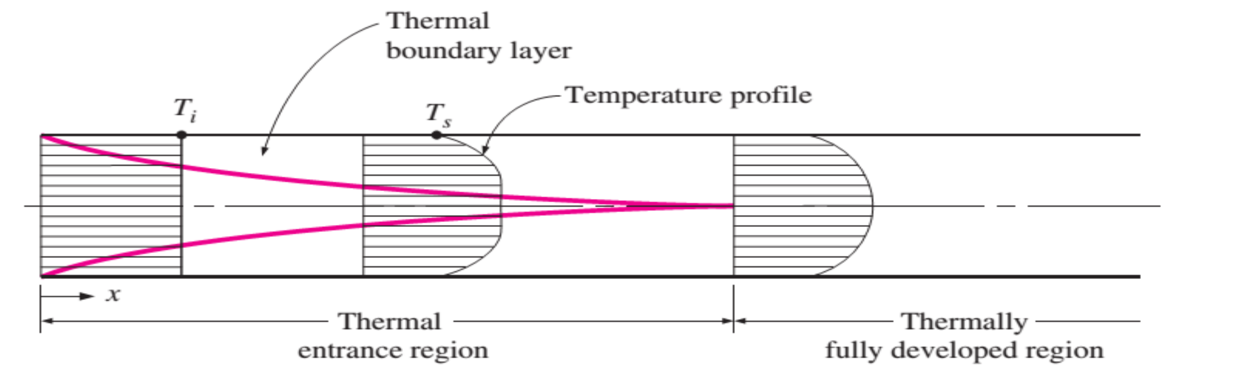
THE ENTRANCE REGION



Hydrodynamic entrance region: from the tube inlet to the point at which the boundary layer merges at the centerline

Hydrodynamically fully developed region: beyond the entrance region in which the velocity profile is fully developed and remains unchanged

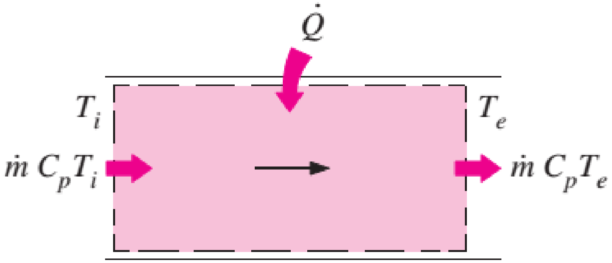
THE ENTRANCE REGION



Thermal entrance region: the thermal boundary layer develops and reaches the tube center

Thermally fully developed region: beyond the thermalentrance region in which the dimensionless temperature profile expressed as $(T_s{-}\mathrm{T})/(T_s{-}T_m)$ remains unchanged

GENERAL THERMAL ANALYSIS

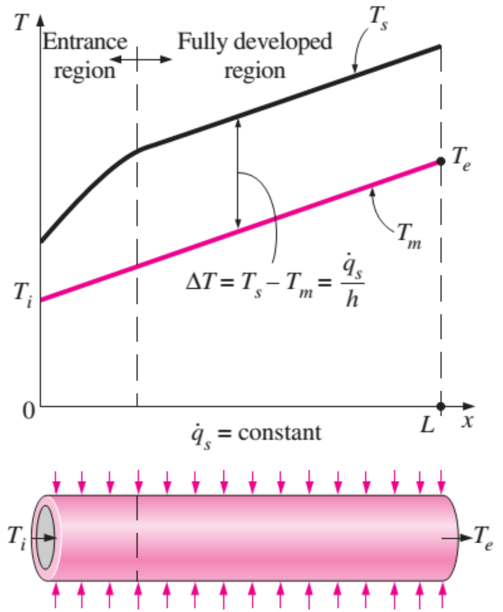


The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

The conservation of energy equation for the steady flow of a fluid in a tube can be expressed as:

$$\dot{Q} = \dot{m} C_p (T_e - T_i)$$

Constant Surface Heat Flux ($\dot{q}_s = constant$)



Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

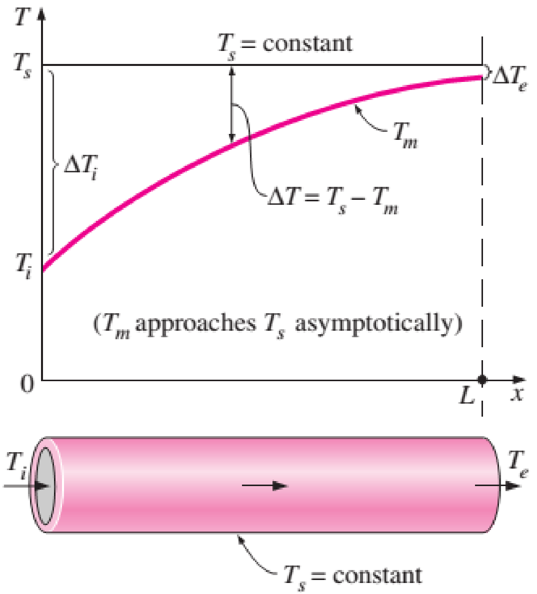
The rate of heat transfer can be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i)$$

The mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

Constant Surface Temperature ($T_s = constant$)



The variation of the *mean fluid* temperature along the tube for the case of constant temperature.

The rate of heat transfer can be expressed as

$$\dot{Q} = hA_s \Delta T_{ave} = hA_s (T_s - T_m)_{ave} \qquad (W)$$

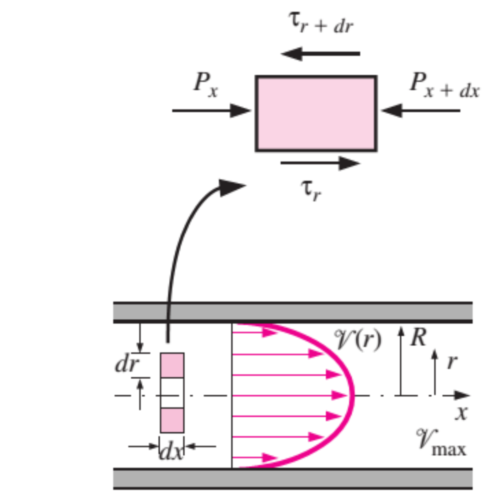
The mean fluid temperature at the tube exit

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

The logarithmic mean temperature difference is a representation of the average temperature difference between the fluid and the surface

$$\Delta T_{ln} = \frac{T_i - T_e}{\ln\left[\frac{T_s - T_e}{T_s - T_i}\right]} = \frac{\Delta T_e - \Delta T_i}{\ln\left[\frac{\Delta T_e}{\Delta T_i}\right]}$$

LAMINAR FLOW IN TUBES



Free body diagram of a cylindrical fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with a horizontal tube in fully developed steady flow.

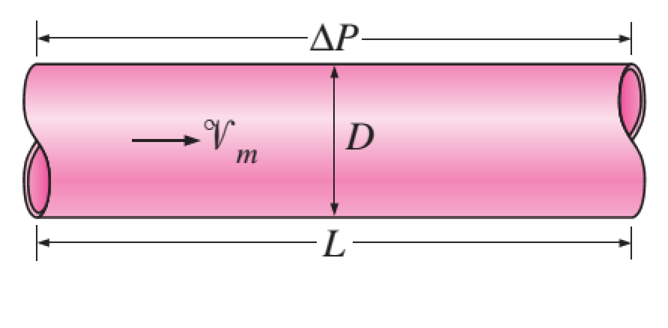
The velocity profile is obtained to be

$$V(r) = 2V_m\left(1 - \frac{r^2}{R^2}\right)$$

The maximum velocity (at the centerline)

$$V_{max} = 2V_m$$

Pressure Drop



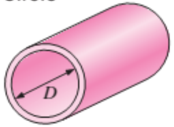
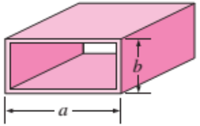
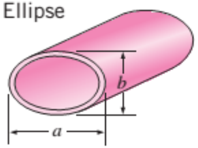
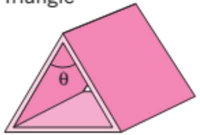
The relation for pressure drop is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and smooth or rough surfaces.

The pressure drop for all types of internal flows (laminar or turbulent flows, circular or noncircular tube smooth or rough surfaces):

$$\Delta P = f \frac{L}{D} \frac{\mu V_m^2}{2}$$

Nusselt Number and Friction Factor

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = \mathcal{V}_m D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number $T_s = \text{Const.}$	Nusselt Number $\dot{q}_s = \text{Const.}$	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel* plates of length L is

$$Nu = 7.54 + \frac{0.03(D_h/L)Re\,Pr}{1 + 0.016[(D_h/L)\,RePr]^{2/3}}$$

TURBULENT FLOW IN TUBES

- The flow in smooth tubes is fully turbulent when $Re > 10{,}000$
- The friction factor in turbulent flow $(10^4 < Re < 10^6)$

$$f = (0.790 \ln Re - 1.64)^{-2}$$

- The Nusselt number in turbulent flow is related to the friction factor

$$Nu = 0.125\, f Re Pr^{1/3}$$

- More accurate Nusselt number using the *second Petukhov equation*

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1)} \qquad \left(\begin{array}{l} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{array}\right)$$

Rough Surfaces

Relative Roughness, ε/L	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6, and are calculated from Eq. 8–73.

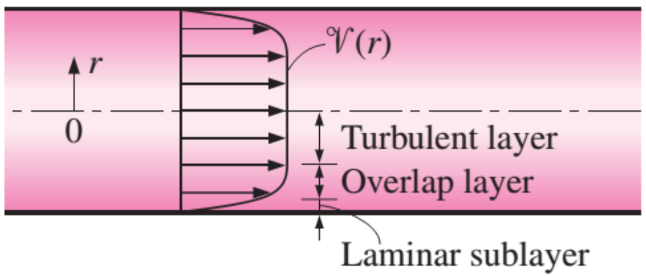
The friction factor is minimum for a smooth pipe and increases with roughness.

The friction factor in fully developed turbulent flow depends on the Reynolds number and the *relative roughness* ε/D *:*

$$\frac{1}{\sqrt{f}} = -2.0 log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

- Tubes with rough surfaces have much higher heat transfer coefficients than smooth surfaces
- It also increases the friction factor and thus the power requirement for the pump or the fan

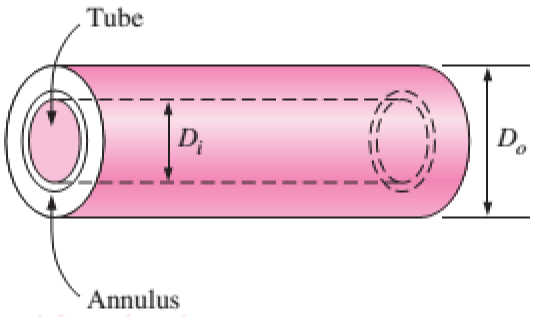
The Velocity Profiles



In turbulent flow, the velocity profile is nearly a straight line in the core region, and any significant velocity gradients occur in the viscous sublayer.

The turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the **hydraulic diameter** $D_h = 4A_c/p$

Flow through Tube Annulus



A double-tube heat exchanger that consists of two concentric tubes.

- The hydraulic diameter of annulus

$$D_h = D_o - D_i$$

- The Nusselt numbers

Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic (Kays and Perkins, Ref. 14)

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

- The convection coefficients for the inner and the outer surfaces can be determined from

$$Nu_i = \frac{h_i D_h}{k} \quad \text{and} \quad Nu_o = \frac{h_o D_o}{k}$$

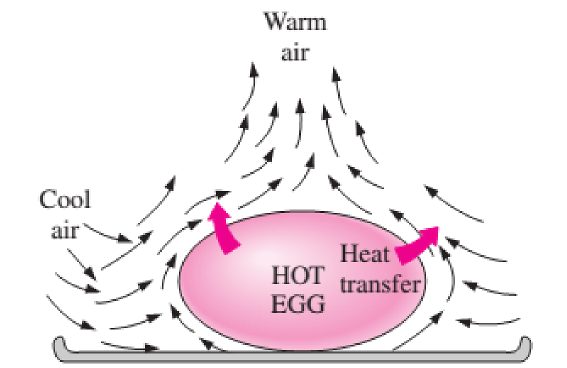
- To improve the accuracy of Nusselt numbers, multiply Nu by the following correction factors:

$$F_i = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \qquad \text{and} \qquad F_o = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16}$$

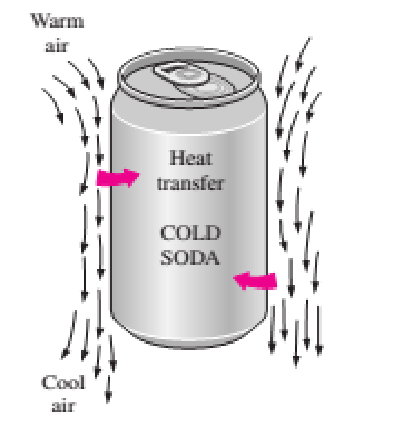
$$(outer\ wall\ adiabatic) \qquad\qquad (inner\ wall\ adiabatic)$$

NATURAL CONVECTION

PHYSICAL MECHANISM



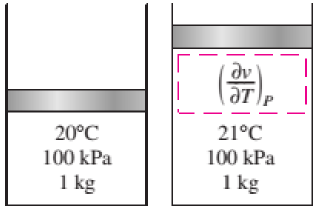
The cooling of a boiled egg in a cooler environment by natural convection.



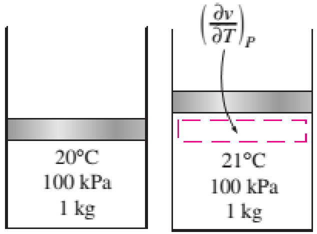
The warming up of a cold drink in a warmer environment by natural convection.

- **Natural convection current** is the motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air
- **Natural convection heat transfer** is a result of natural convection current

- **volume expansion coefficient β** is the property that represents the variation of the density of a fluid with temperature at constant pressure



(*a*) A substance with a large β



(*b*) A substance with a small β

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

- The volume expansion coefficient can be expressed approximately by:

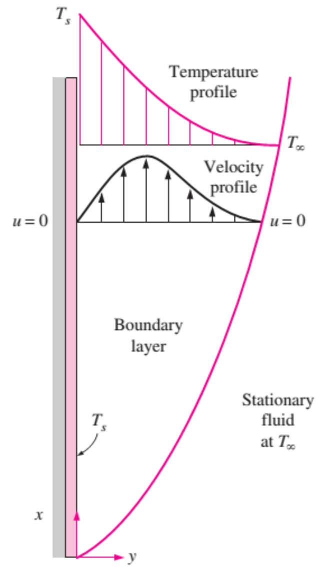
$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \quad \text{at } P_{const}$$

ρ_∞ and T_∞ is the density and temperature of the quiescent fluid away from the surface.

- The volume expansion coefficient of an ideal gas:

$$\beta_{ideal\ gas} = \frac{1}{T} \quad (1/K)$$

EQUATION OF MOTION

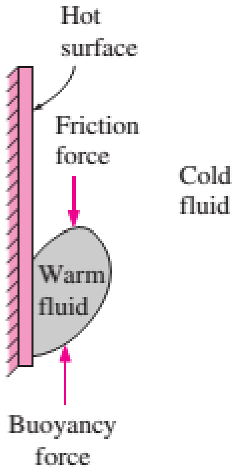


Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

- The thickness of the boundary layer increases in the flow direction
- The fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface
- The equation that governs the fluid motion in the boundary layer due to the effect of buoyancy

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

THE GRASHOF NUMBER



The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

Grashof Number Gr_L is dimensionless parameter that represents the natural convection effects

$$Gr_L = \frac{g\beta(T_S - T_\infty)L_c^3}{v^2}$$

g = gravitational acceleration

β = coefficient of volume expansion

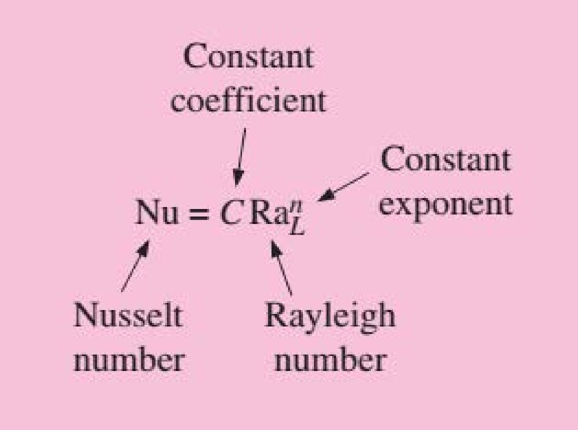
T_s = temperature of the surface

T_∞ = temperature of the fluid sufficiently far from the surface

L_c = characteristic length of the geometry

v = kinematic viscosity of the fluid

NATURAL CONVECTION OVER SURFACES



Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant *n* multiplied by another constant *C*, both of which are determined experimentally.

- The simple empirical correlations for the average Nusselt number Nu in natural convection:

$$Nu = \frac{h\,L_c}{k} = C(Gr_L\,Pr)^n = C Ra_L^n$$

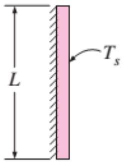
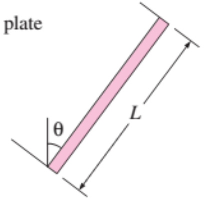
- **Rayleigh number** is the product of the Grashof and Prandtl numbers:

$$Ra_L = Gr_L\,Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{v^2}\,\mathrm{Pr}$$

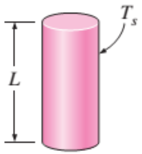
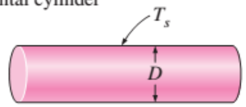
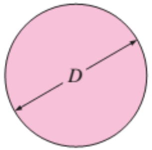
- The rate of heat transfer by natural convection:

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad (W)$$

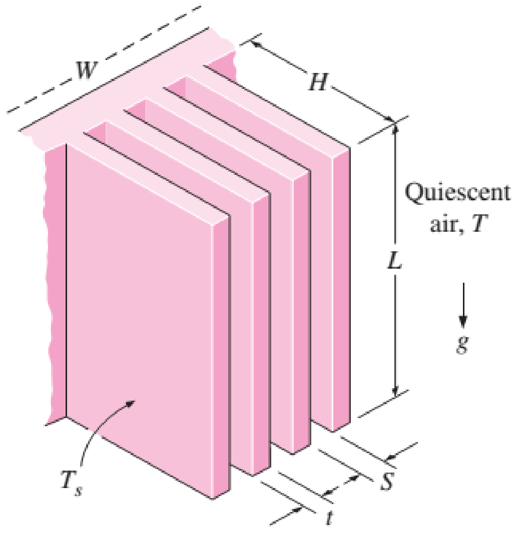
NUSSELT NUMBER FOR NATURAL CONVECTION OVER SURFACES

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4}$ (9-19) $\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3}$ (9-20) $\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $\mathrm{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\mathrm{Nu} = 0.54\mathrm{Ra}_L^{1/4}$ (9-22) $\mathrm{Nu} = 0.15\mathrm{Ra}_L^{1/3}$ (9-23) $\mathrm{Nu} = 0.27\mathrm{Ra}_L^{1/4}$ (9-24)

NUSSELT NUMBER FOR NATURAL CONVECTION OVER SURFACES

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)

Natural Convection Cooling of Finned Surfaces



Various dimensions of a finned surface oriented vertically.

- The Rayleigh number:

$$Ra_s = \frac{g\beta(T_s - T_\infty)S^3}{v^2}\Pr \quad \text{and} \quad Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{v^2}\Pr = Ra_s\frac{L^3}{S^3}$$

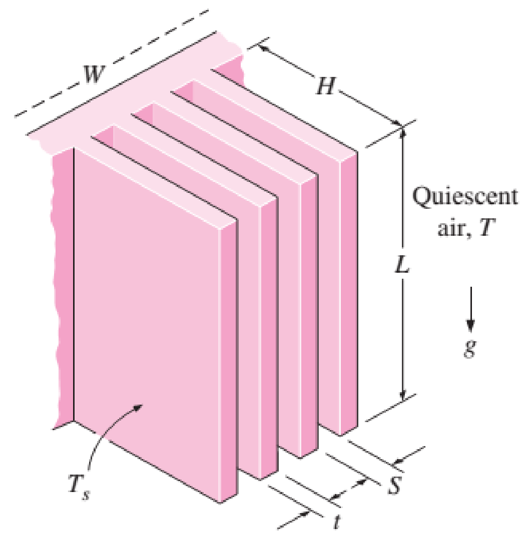
- The average Nusselt number:

$$Nu = \frac{h\,S}{k} = \left[\frac{576}{\left(\frac{Ra_S S}{L}\right)^2} + \frac{2.873}{\left(\frac{Ra_S S}{L}\right)^{0.5}}\right]^{-0.5}$$

- The optimum fin spacing

$$S_{opt} = 2.714\left(\frac{S^3 L}{Ra_s}\right)^{0.25} = 2.714\frac{L}{Ra_L^{0.25}}$$

Natural Convection Cooling of Finned Surfaces



Various dimensions of a finned surface oriented vertically.

- The Nusselt number is a constant when $S = S_{opt}$:

$$Nu = \frac{h\,S_{opt}}{k} = 1.307$$

- The rate of heat transfer :

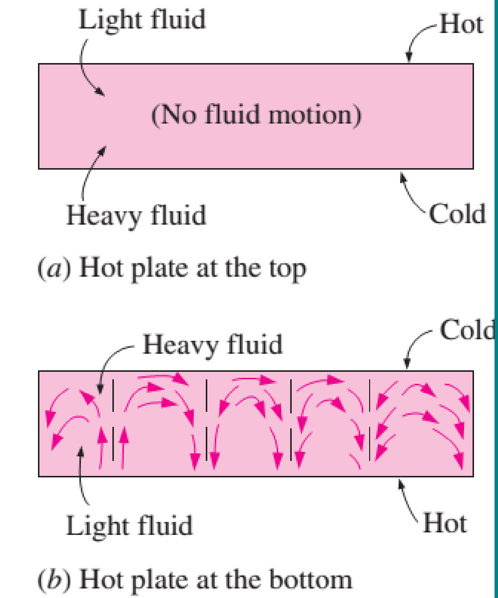
$$\dot{Q} = h(2nLH)(T_s - T_\infty)$$

Where $n = W/(S+t)$ is the number of fins

- All fluid properties are evaluated at T_{ave}

$$T_{ave} = (T_s + T_\infty)/2$$

NATURAL CONVECTION INSIDE ENCLOSURES

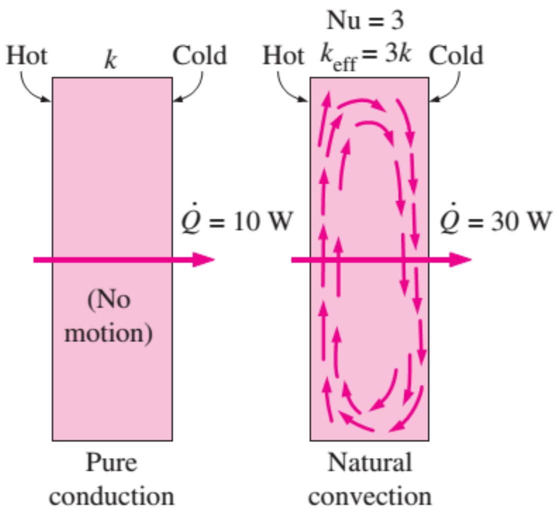


Convective currents in a horizontal enclosure with (a) hot plate at the top and (b) hot plate at the bottom.

- When the hotter plate is at the top, no convection currents will develop in the enclosure.
- When the hotter plate is at the bottom, the heavier fluid will be on top of the lighter fluid.
- The Rayleigh number for an enclosure is determined from

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{v^2}\,\mathrm{Pr}$$

Effective Thermal Conductivity



A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction.*

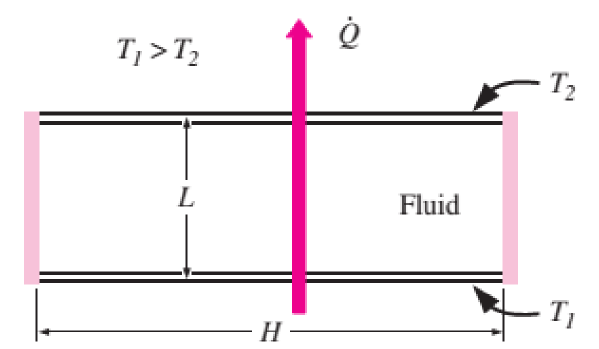
- The rate of heat transfer through the enclosure:

$$\dot{Q} = hA_s(T_1 - T_2) = kNuA_s\frac{T_1 - T_2}{L_c}$$

- Effective thermal conductivity

$$k_{eff} = kNu$$

Horizontal Rectangular Enclosures



A horizontal rectangular enclosure with isothermal surfaces.

- For horizontal enclosures that contain air

$$Nu = 0.195 Ra_L^{1/4}, \text{ for } 10^4 < Ra_L < 4\text{x}10^5$$

$$Nu = 0.068 Ra_L^{1/3}, \text{ for } 4\text{x}10^5 < Ra_L < 10^7$$

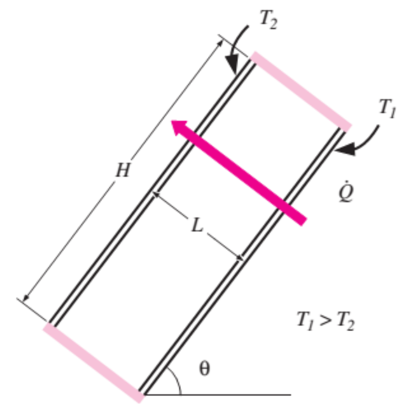
- For other gases with $0.5 < Pr < 2$

$$Nu = 0.069 Ra_L^{1/3} Pr^{0.074}, \text{ for } 3\text{x}10^5 < Ra_L < 7\text{x}10^9$$

- For horizontal enclosures

$$Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L}\right]^{+} + \left[\frac{Ra_L^{1/3}}{18} - 1\right]^{+}$$

Inclined Rectangular Enclosures



An inclined rectangular enclosure with isothermal surfaces.

- For large aspect ratios $(H/L \geq 12)$

$$Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L\, cos\theta}\right]^{+}\left(1 - \frac{1708(sin1.8\theta)^{1.6}}{Ra_L\, cos\theta}\right) + \left[\frac{(Ra_L\, cos\theta)^{1/3}}{18} - 1\right]^{+}$$

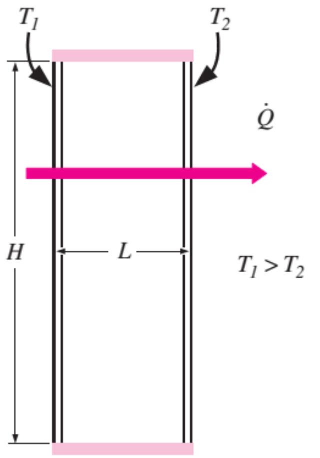
- For enclosures with smaller aspect ratios $(H/L < 12)$

$$Nu = Nu_{\theta=0^\circ}\left(\frac{Nu_{\theta=90^\circ}}{Nu_{\theta=0^\circ}}\right)^{\theta/\theta cr}(sin\theta_{cr})^{\theta/(4\theta_{cr})} \qquad 0^\circ < \theta < \theta_{cr}$$

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
> 12	70°

Vertical Rectangular Enclosures



A vertical rectangular enclosure with isothermal surfaces.

- For vertical enclosures

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29} \qquad \begin{array}{c} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L\,\mathrm{Pr}/(0.2+\mathrm{Pr}) > 10^3 \end{array}$$

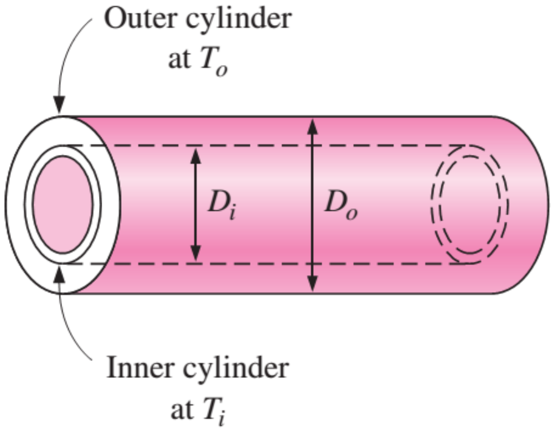
$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{array}{c} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L < 10^{10} \end{array}$$

- For vertical enclosures with larger aspect ratios

$$\mathrm{Nu} = 0.42\,\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} \qquad \begin{array}{c} 10 < H/L < 40 \\ 1 < \mathrm{Pr} < 2\times 10^4 \\ 10^4 < \mathrm{Ra}_L < 10^7 \end{array}$$

$$\mathrm{Nu} = 0.46\mathrm{Ra}_L^{1/3} \qquad \begin{array}{c} 1 < H/L < 40 \\ 1 < \mathrm{Pr} < 20 \\ 10^6 < \mathrm{Ra}_L < 10^9 \end{array}$$

Concentric Cylinders



Two concentric horizontal isothermal cylinders.

- The rate of heat transfer

$$\dot{Q} = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o) \qquad (\mathrm{W/m})$$

- The effective thermal conductivity

$$\frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861+Pr}\right)^{1/4}\left(F_{cyl}Ra_L\right)^{1/4}$$

- The geometric factor for concentric cylinders

$$F_{cyl} = \frac{\left[\ln(D_o/D_i)\right]^4}{L_c^3\left(D_i^{-3/5}+D_o^{-3/5}\right)^5}$$

where $L_c = (D_o - D_i)/2$