

Heat Transfer: Flow Across Tube Banks

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Introduction

Tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners.

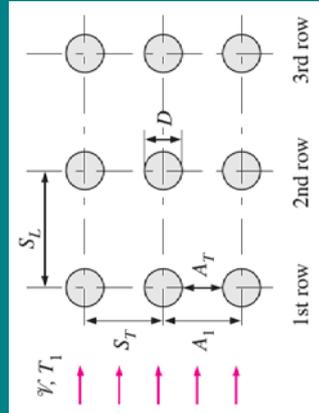
The tubes are usually placed in a shell (and thus the name shell-and-tube heat exchanger), especially when the fluid is a liquid, and the fluid flows through the space between the tubes and the shell.



ANALYSIS OF FLOW ACROSS TUBE BANKS

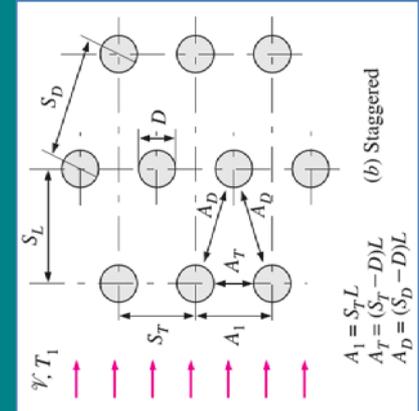
The tubes in a tube bank are usually arranged either in-line or staggered in the direction of flow

in-line



Flow
direction

staggered



The arrangement of the tubes in the tube bank is characterized by the *transverse pitch* S_T , *longitudinal pitch* S_L , and the *diagonal pitch* S_D between tube centers.

ANALYSIS OF FLOW ACROSS TUBE BANKS

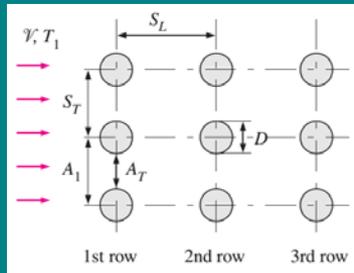
In tube banks, the flow characteristics are dominated by *the maximum velocity* V_{max} that occurs within the tube bank rather than the approach velocity V . So, the Reynolds number is defined on the basis of maximum velocity as:

$$Re_D = \frac{\rho V_{max} D}{\mu} = \frac{V_{max} D}{\nu}$$

The maximum velocity is determined from the conservation of mass requirement for steady incompressible flow.

- In-line arrangement

The maximum velocity occurs at the minimum flow area between the tubes.



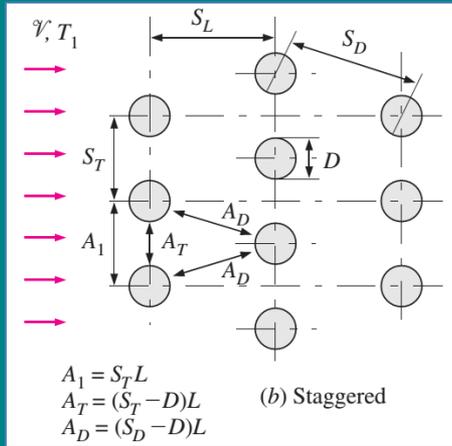
$\rho V A_1 = \rho V_{max} A_T$ or $V S_T = V_{max} (S_T - D)$
then the maximum velocity becomes:

$$V_{max} = \frac{S_T}{S_T - D} V$$



ANALYSIS OF FLOW ACROSS TUBE BANKS

- Staggered arrangement



The fluid approaching through area A_1 passes through area A_T and then through area $2A_D$ as it wraps around the pipe in the next row.

- If $2A_D > A_T$, maximum velocity will still occur at A_T between the tubes, thus the maximum velocity for this case uses V_{max} for in-line arrangement
- If $2A_D < A_T$ [or, if $2(S_D - D) < (S_T - D)$] maximum velocity will occur at the diagonal cross sections, and

the maximum velocity becomes

Staggered and $S_D < (S_T + D)/2$:

$$V_{max} = \frac{S_T}{2(S_D - D)} V$$

since $\rho V A_1 = \rho V_{max} (2A_D)$ or $V S_T = V_{max} (S_D - D)$



ANALYSIS OF FLOW ACROSS TUBE BANKS

Flow through tube banks is studied experimentally since it is too complex to be treated analytically.

Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. More recently, **Zukauskas** has proposed correlations whose general form is

$$Nu_D = \frac{hD}{k} = C Re_D^m Pr^n \left(\frac{Pr}{Pr_s} \right)^{0.25}$$

the values of the constants C , m , and n depend on value Reynolds number.



ANALYSIS OF FLOW ACROSS TUBE BANKS

Such correlations are given in this table explicitly for $0.7 < Pr < 500$ and $0 < Re_D < 2 \times 10^6$ for tube banks with 16 or more rows

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9 Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52 Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000– 2×10^5	$Nu_D = 0.27 Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2×10^5 – 2×10^6	$Nu_D = 0.033 Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04 Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71 Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000– 2×10^5	$Nu_D = 0.35 (S_T/S_L)^{0.2} Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2×10^5 – 2×10^6	$Nu_D = 0.031 (S_T/S_L)^{0.2} Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

The arithmetic mean of the inlet and outlet temperatures

$$T_m = \frac{T_i + T_o}{2}$$

The uncertainty in the values of Nusselt number obtained from these relations is ± 15 percent. Note that all properties except Pr_s are to be evaluated at the arithmetic mean temperature of the fluid

ANALYSIS OF FLOW ACROSS TUBE BANKS

For tube banks with N_L provided that they are modified as

$$Nu_{D,N_L} = F Nu_D$$

F is a correction factor from the table

Correction factor F to be used in $Nu_{D,N_L} = F Nu_D$ for $N_L < 16$ and $Re_D > 1000$
(from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

For $Re_D > 1000$, the correction factor is independent of Reynolds number.



ANALYSIS OF FLOW ACROSS TUBE BANKS

Then the heat transfer rate can be determined from Newton's law of cooling using a suitable temperature difference ΔT . Temperature difference can be defined by *the logarithmic mean temperature difference* ΔT_{ln}

$$\Delta T_{ln} = \frac{(T_s - T_o) - (T_s - T_i)}{\ln \left[\frac{(T_s - T_o)}{(T_s - T_i)} \right]} = \frac{\Delta T_o - \Delta T_i}{\ln \left[\frac{\Delta T_o}{\Delta T_i} \right]}$$

the outlet temperature of the fluid T_o be determined from

$$T_o = T_s - (T_s - T_i) \exp \left(- \frac{A_s h}{\dot{m} C_p} \right)$$

$A_s = N\pi DL$ is the heat transfer surface area

$\dot{m} = \rho V(N_T S_T L)$ is the mass flow rate of the fluid

N is the total number of tubes in the bank

N_T is the number of tubes in a transverse plane

L is the length of the tubes

V is the velocity of the fluid just before entering the tube bank



ANALYSIS OF FLOW ACROSS TUBE BANKS

After we get *the logarithmic mean temperature difference* ΔT_{ln} , now we can determine the heat transfer rate from

$$\dot{Q} = hA_s\Delta T_{ln} = \dot{m}C_p(T_o - T_i)$$

The second relation is usually more convenient to use since it does not require the calculation of ΔT_{ln}



PRESSURE DROP

Another quantity of interest associated with tube banks is the pressure drop ΔP , which is the difference between the pressures at the inlet and the outlet of the tube bank. It can be expressed as:

$$\Delta P = N_L f x \frac{\rho V_{max}^2}{2}$$

N_L is the number of rows

f is the friction factor

x is the correction factor

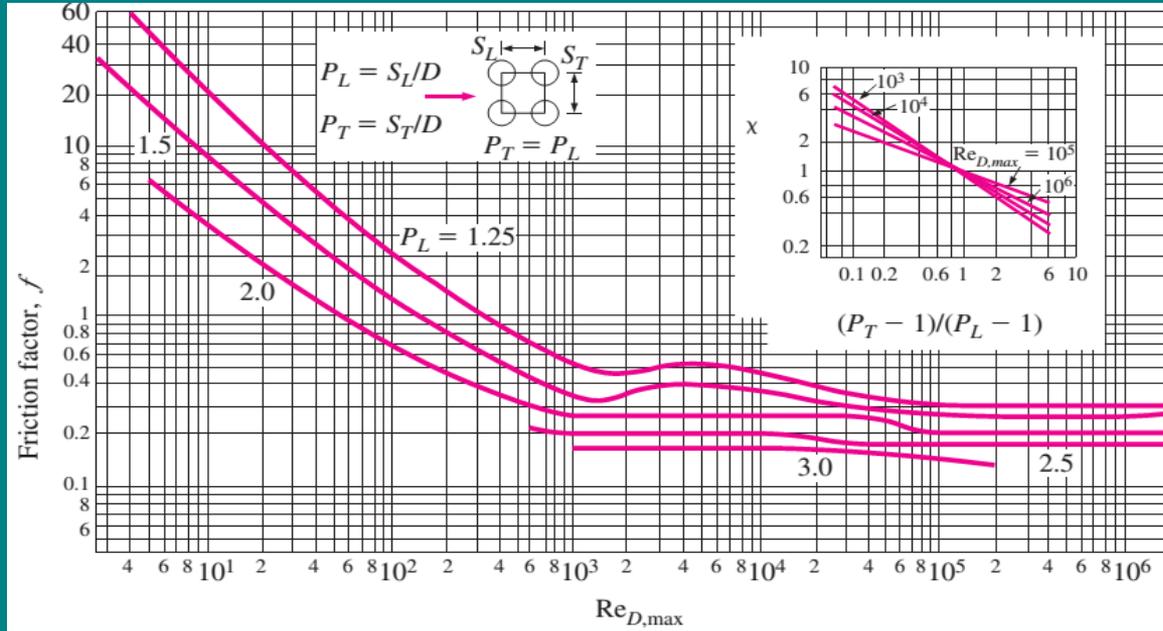
f and x are plotted against the Reynolds number based on the maximum Velocity V_{max}



PRESSURE DROP

- In-line arrangement

The friction factor in this graphic is for a square in-line tube bank ($S_T = S_L$)



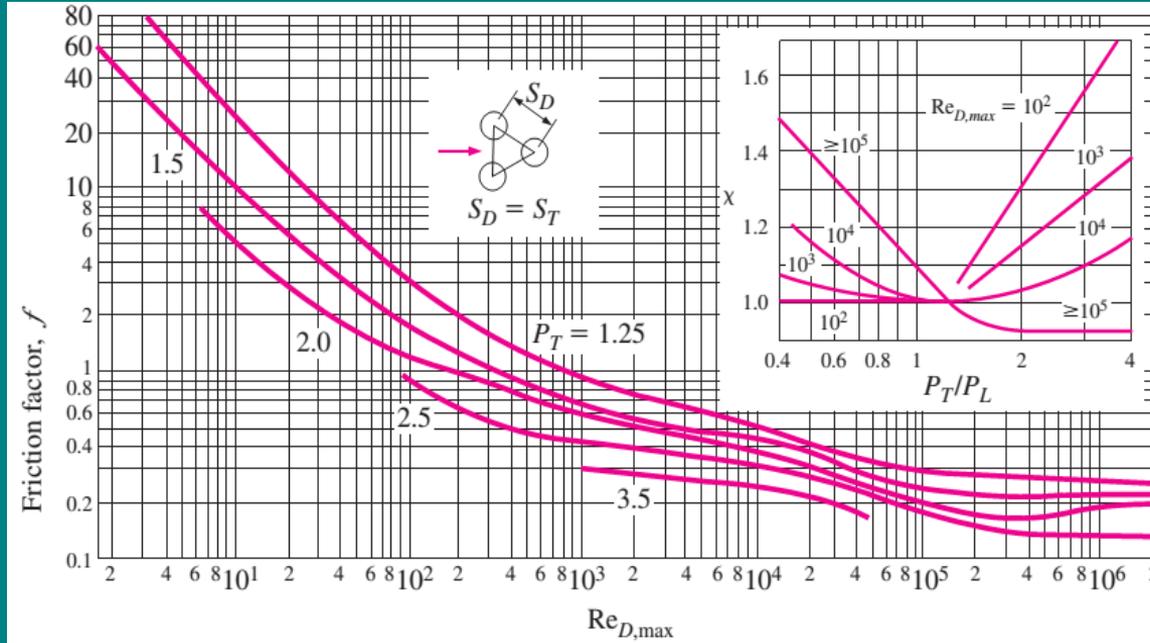
$x = 1$
for both square
and equilateral
triangle
arrangements.

The correction factor given in the insert is used to account for the effects of deviation of rectangular in-line arrangements from square arrangement.

PRESSURE DROP

- Staggered arrangement

This is used for an equilateral staggered tube bank ($S_T = S_D$)



$x = 1$
for both square
and equilateral
triangle
arrangements.

The correction factor is to account for the effects of deviation from equilateral arrangement.

PUMPING POWER

When the pressure drop is available, the pumping power required can be determined from:

$$\dot{W}_{pump} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho}$$

$\dot{V} = V(N_T S_T L)$ is the volume flow rate

$\dot{m} = \rho\dot{V} = \rho V(N_T S_T L)$ is the mass flow rate of the fluid through the tube bank

The power required to keep a fluid flowing through the tube bank (and thus the operating cost) is proportional to the pressure drop.



EXAMPLE

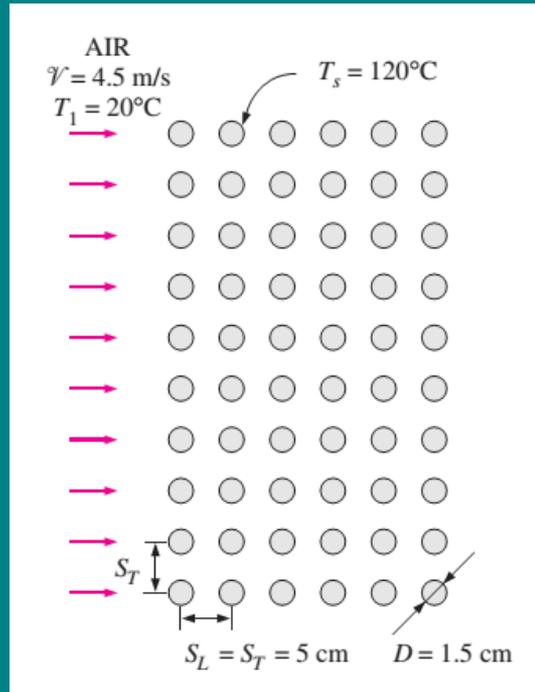
Preheating Air by Geothermal Water in a Tube Bank

In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120°C flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20°C and 1 atm with a mean velocity of 4.5 m/s and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 5\text{cm}$. There are 6 rows in the flow direction with 10 tubes in each row. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.



EXAMPLE

Preheating Air by Geothermal Water in a Tube Bank



Assumptions: 1. Steady operating conditions exist. 2. The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties: The air properties at the assumed mean temperature of 60°C and 1 atm are Table A-15:

$$\begin{aligned} k &= 0.02808 \text{ W/m} \cdot \text{K}, & \rho &= 1.06 \text{ kg/m}^3 \\ C_p &= 1.007 \text{ kJ/kg} \cdot \text{K}, & Pr &= 0.7202 \\ \mu &= 2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s} & Pr_s &= Pr_{@T_s} = 0.7073 \end{aligned}$$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204 \text{ kg/m}^3$



EXAMPLE

Preheating Air by Geothermal Water in a Tube Bank

Analysis: $D = 0.015$ m, $S_L = S_T = 0.05$ cm, $V = 4.5$ m/s

$$V_{max} = \frac{S_T}{2(S_D - D)} V = \frac{0.05}{2(0.05 - 0.015)} (4.5 \text{ m/s}) = 6.43 \text{ m/s}$$

$$Re_D = \frac{\rho V_{max} D}{\mu} = \frac{(1.06 \frac{\text{kg}}{\text{m}^3})(6.43 \frac{\text{m}}{\text{s}})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 5091$$

From the table, for in-line arrangement and $Re_D = 5091$, Nusselt number is

$$\begin{aligned} Nu_D &= 0.27 Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25} \\ &= 0.27(5091)^{0.63} (0.7202)^{0.36} (0.7202/0.7073)^{0.25} = 52.2 \end{aligned}$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case $N_L = 6$, so we have to use correction factor from the table. $F = 0.945$

$$Nu_{D,N_L} = F Nu_D = (0.945)(52.2) = 49.3$$

$$h = \frac{Nu_{D,N_L} D}{D} = \frac{49.3(0.02808 \frac{\text{W}}{\text{m}} \cdot ^\circ\text{C})}{0.015 \text{ m}} = 92.2 \text{ W/m}^2 \cdot ^\circ\text{C}$$



EXAMPLE

Preheating Air by Geothermal Water in a Tube Bank

Analysis: Total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$

The heat transfer surface area (for a unit tube length $L = 1 \text{ m}$) :

$$A_S = N\pi DL = 60\pi(0.015 \text{ m})(1 \text{ m}) = 2.827 \text{ m}^2$$

The mass flow rate of air (evaluated at the inlet)

$$\begin{aligned}\dot{m} &= \dot{m}_1 = \rho_1 V(N_T S_T L) \\ &= (1.204 \text{ kg/m}^3)(4.5 \text{ m/s})(10)(0.05 \text{ m})(1 \text{ m}) = 2.709 \text{ kg/s}\end{aligned}$$

The fluid outlet temperature

$$\begin{aligned}T_o &= T_s - (T_s - T_i) \exp\left(-\frac{A_S h}{\dot{m} C_p}\right) \\ &= 120 - (120 - 20) \exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2 \cdot \text{°C})}{(2.709 \text{ kg/s})(1.007 \text{ kJ/kg} \cdot \text{K})}\right) = 29.11 \text{ °C}\end{aligned}$$



EXAMPLE

Preheating Air by Geothermal Water in a Tube Bank

Analysis: The log mean temperature difference:

$$\Delta T_{ln} = \frac{(T_s - T_o) - (T_s - T_i)}{\ln \left[\frac{(T_s - T_o)}{(T_s - T_i)} \right]} = \frac{(120 - 29.11) - (120 - 20)}{\ln \left[\frac{(120 - 29.11)}{(120 - 20)} \right]} = 95.4 \text{ } ^\circ\text{C}$$

The rate of heat transfer

$$\dot{Q} = hA_s \Delta T_{ln} = (92.2 \text{ W/m}^2 \cdot ^\circ\text{C})(2.827 \text{ m}^2)(95.4 \text{ } ^\circ\text{C}) = 2.49 \times 10^4 \text{ W}$$

or

$$\dot{Q} = \dot{m}C_p(T_o - T_i) = (2.709 \text{ kg/s})(1.007 \text{ kJ/kg} \cdot \text{K})(29.11 - 20)^\circ\text{C} = 2.49 \times 10^4 \text{ W}$$



EXAMPLE

Preheating Air by Geothermal Water in a Tube Bank

Analysis: Then the pressure drop across the tube bank, corresponding to $Re_D = 5091$ and $S_L/D = 5/1.5 = 3.33$, we get from the graphic $f = 0.16$ and $x = 1$ for the square arrangements.

$$\begin{aligned}\Delta P &= N_L f x \frac{\rho V_{max}^2}{2} \\ &= 6(0.16)(1) \frac{\left(1.06 \frac{kg}{m^3}\right) \left(6.43 \frac{m}{s}\right)^2}{2} = 21 \text{ Pa}\end{aligned}$$

