

*Bandung Train The Trainers 16<sup>th</sup> -28<sup>th</sup> May 2016*

# Reservoir Geomechanics

Train The Trainers workshop , WP 1.04 WP leader Peter Fokker TNO

*Auke Barnhoorn, Peter Fokker*

# Cooperating Companies and Universities



INAGA



IF Technology



DNVGL



Institute Teknologi Bandung



Delft University of Technology  
Department of Geo-Technology



University of Twente  
Faculty of ITC



Universitas Gadjah Mada



Universitas Indonesia



University of Utrecht  
Faculty of Geosciences –  
Department of Earth Sciences



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# Reservoir Geomechanics Module

- **Half a day** lecture material containing a basic introduction into reservoir geomechanics targeted to MSc student community.
- No prior knowledge of geomechanics is required to attend this course.
- Material contain powerpoint lecture (~3 x 40 min) + multiple in-class exercises (2x 10 minutes in between class and 2 exercises for the end of the day).
- At the end of the module students will have a better understanding of the basics of geomechanics and an appreciation of its importance for subsurface engineering for geothermal and petroleum reservoirs.

# Importance of reservoir geomechanics

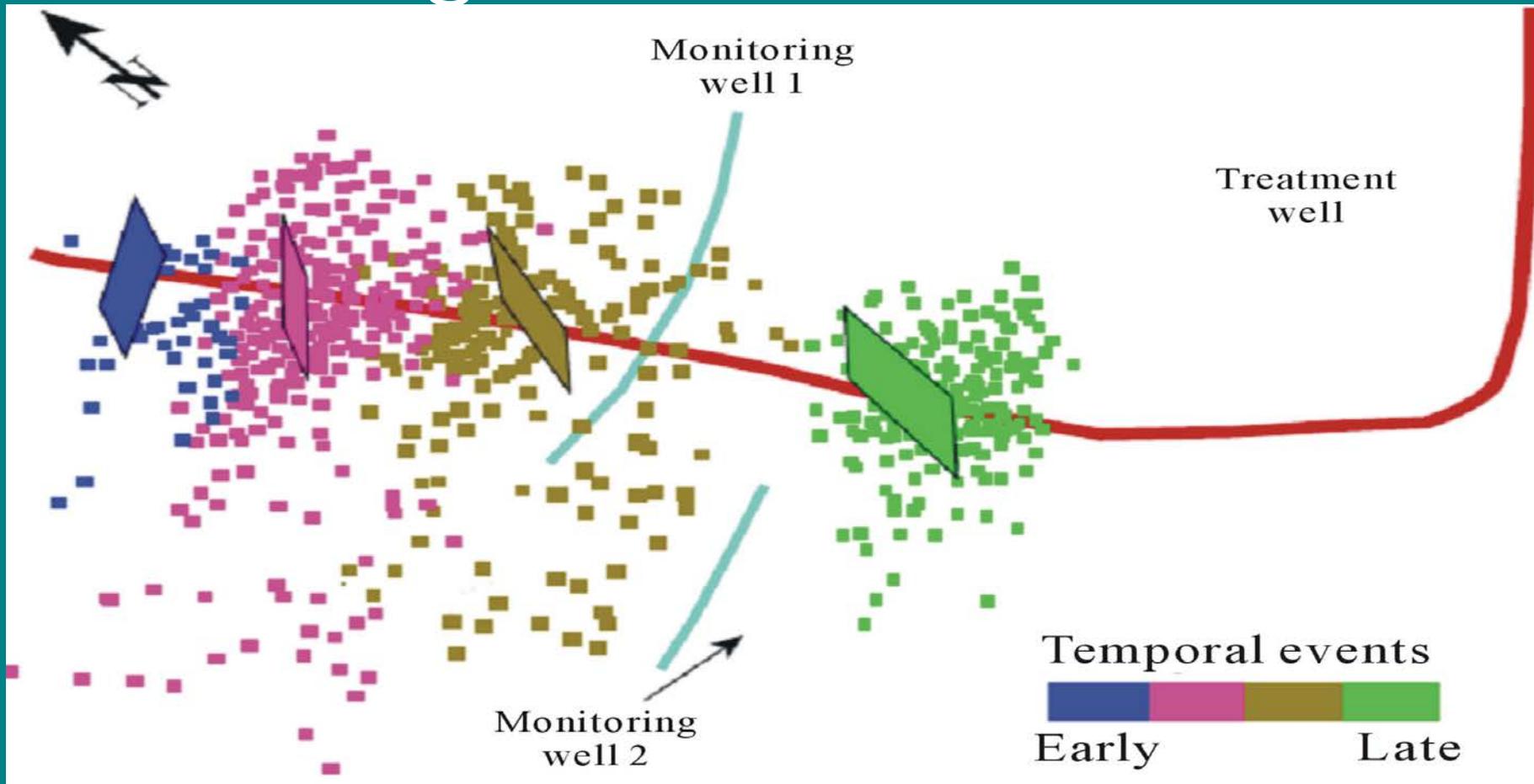
- Seismicity (induced and natural)
- Hydraulic fracturing
  - Borehole fluid pressures
  - Fracture orientation
- Well bore stability
- Compaction due to production
- Current stress field in the subsurface
- Geophysics – seismic exploration
- Etc.

# M6.0 April 6<sup>th</sup>, 2016



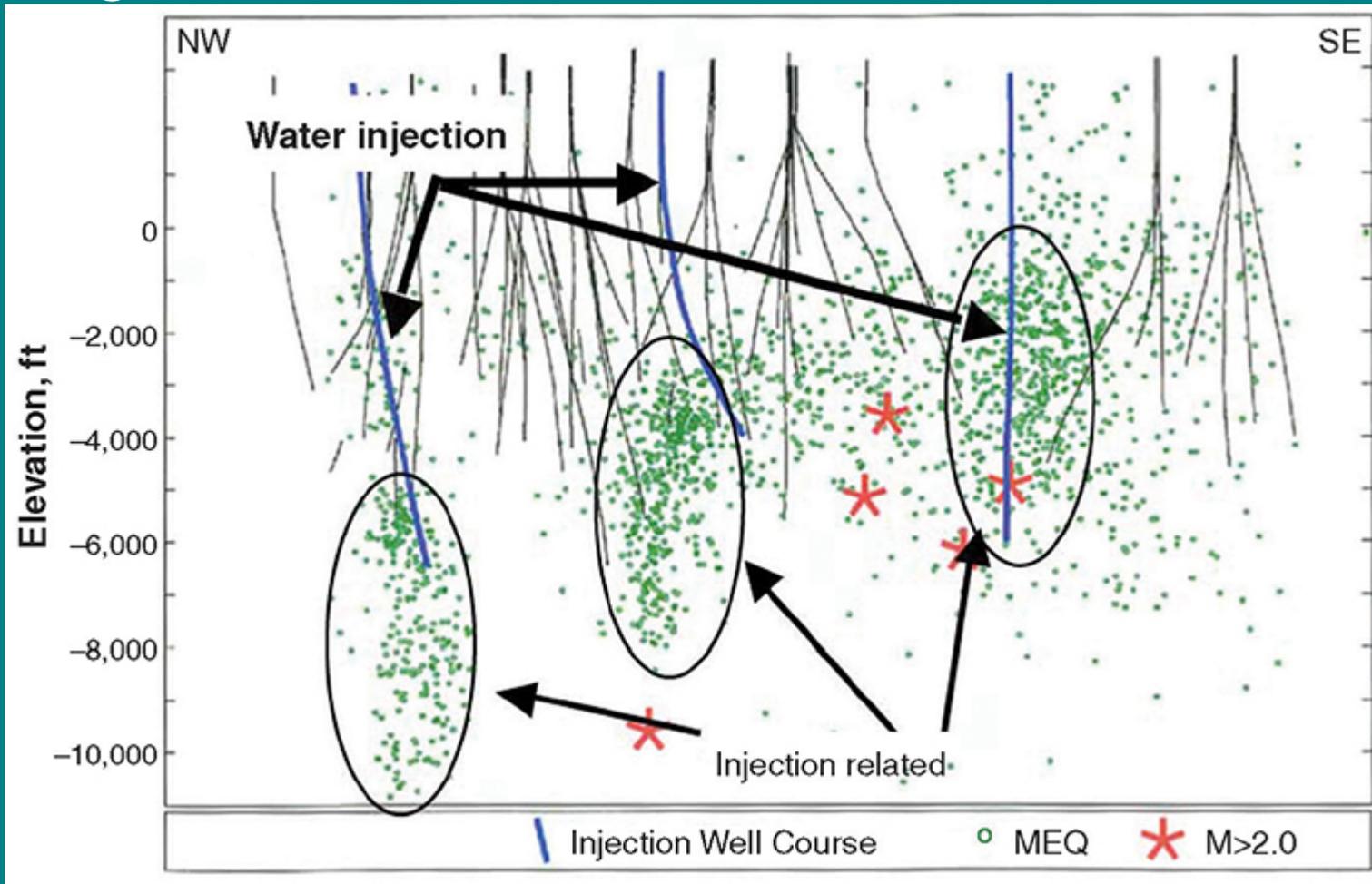
<http://thewatchers.adorraeli.com/2016/04/06/strong-and-shallow-m6-0-earthquake-hit-near-the-coast-of-java-indonesia/>

# Induced seismicity due to hydraulic fracturing



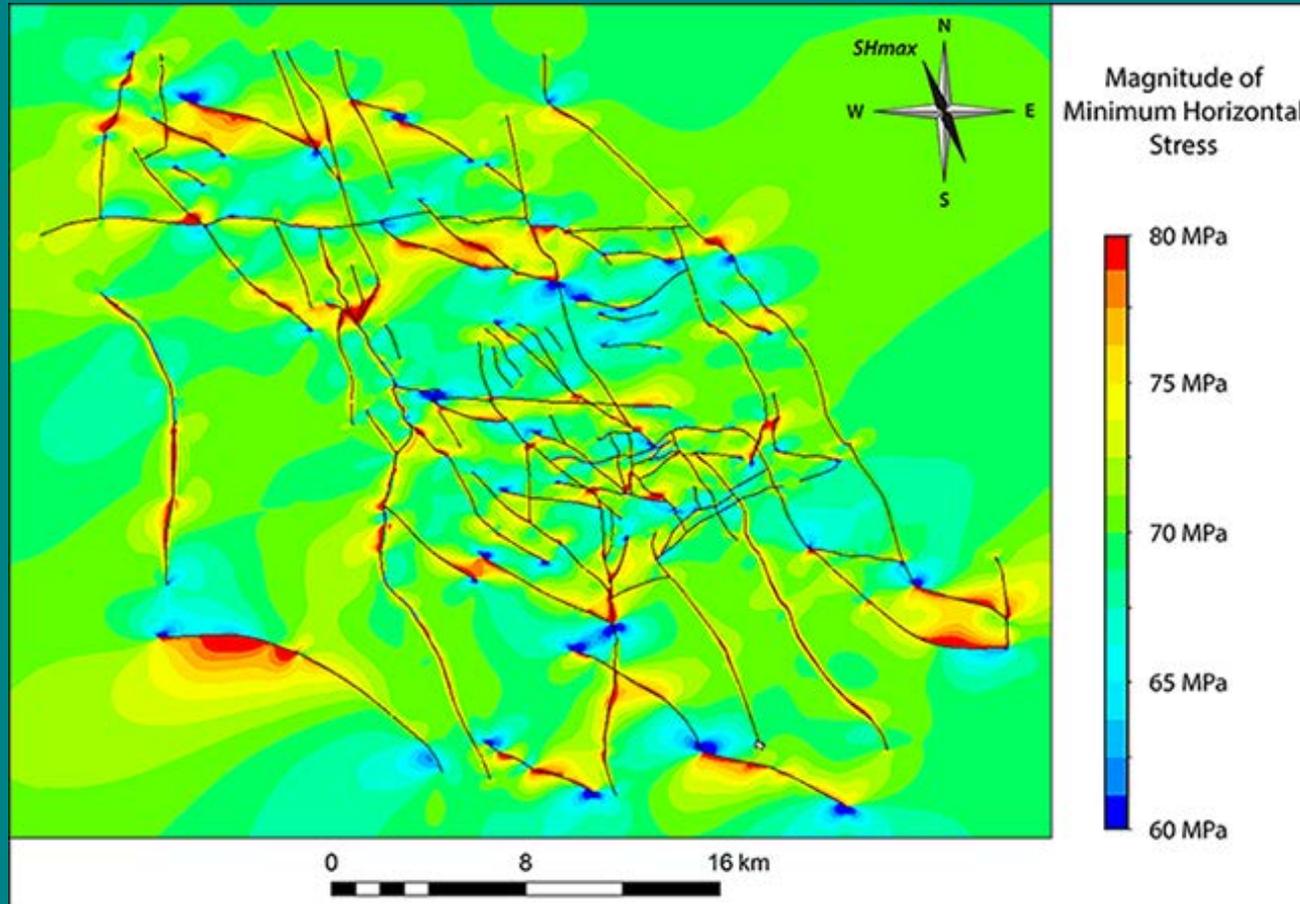
Abdulaziz, 2013

# Induced seismicity due waste water injection



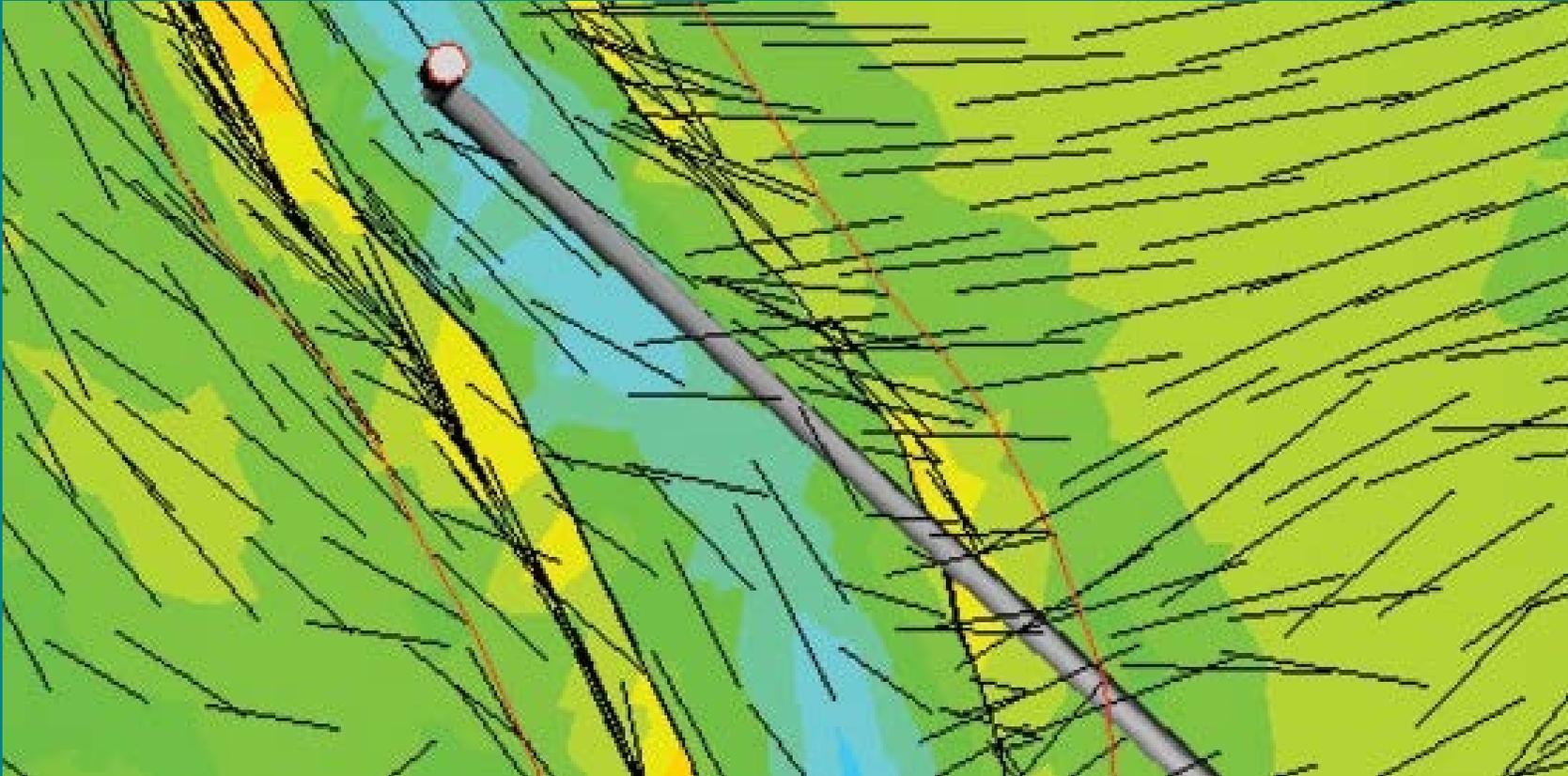
<http://www.spe.org/jpt/article/7139-searching-for-solutions-to-induced-seismicity/>

# Stress distribution in a reservoir - magnitude



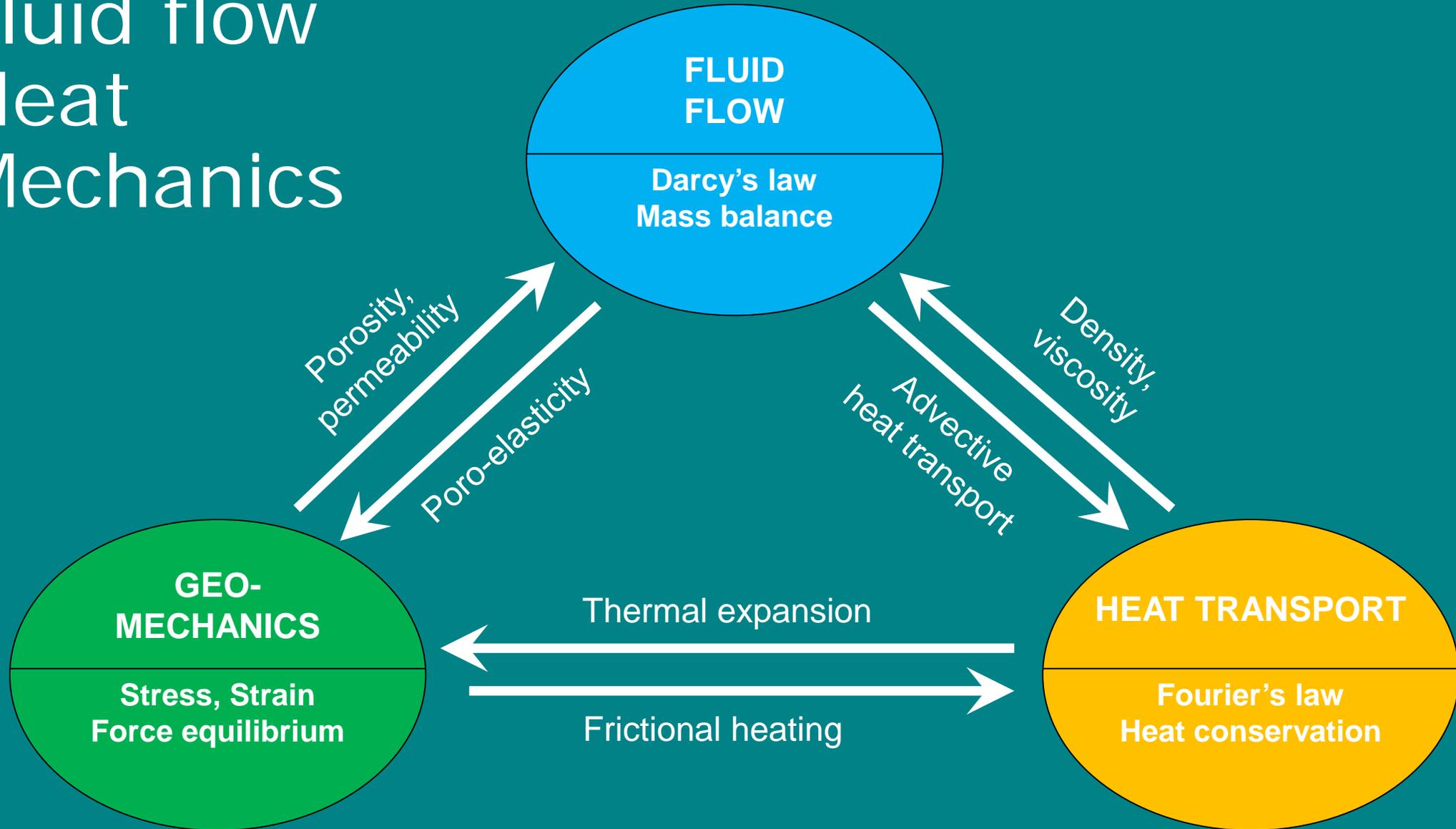
[http://www.geo.tu-darmstadt.de/fg/inggeol/inggeo\\_forschung/dgmk\\_forschungsprojekt\\_721/dgmk\\_project\\_721.en.jsp](http://www.geo.tu-darmstadt.de/fg/inggeol/inggeo_forschung/dgmk_forschungsprojekt_721/dgmk_project_721.en.jsp)

# Stress distribution in a reservoir - orientation



[http://www.golder.com/ph/modules.php?name=Newletters&op=viewarticle&p\\_id=167&page\\_id=1100&article\\_id=570](http://www.golder.com/ph/modules.php?name=Newletters&op=viewarticle&p_id=167&page_id=1100&article_id=570)

# Fluid flow Heat Mechanics



From Peter Fokker, TNO

# Use of geomechanics for geothermals

- Knowledge of the orientation of the natural faults/fractures
- Hydraulic stimulation to improve fluid path ways
- Fluid flow/production may influence PT conditions in the subsurface and/or porosity/permeability of the rocks that may result in geomechanical responses

# Outline for this afternoon

- Basics of geomechanics
  - Stress
  - Strain
  - Elasticity
  - Inelasticity (fracturing, creep, ductile flow)
- Subsurface stresses & rock failure in wellbores
- Application examples of reservoir geomechanics
  
- Exercises

# Stress

- Stress is everywhere in the Earth
- Stress varies with depth, location, locality, tectonic setting etc.
  
- Stress controls dynamics in the Earth
  - Movement of rocks on geological timescales
  - Rock shape and volume changes
  - Failure of rocks

# Stress

- $Stress = \frac{Force}{Area} = \frac{F}{A}$

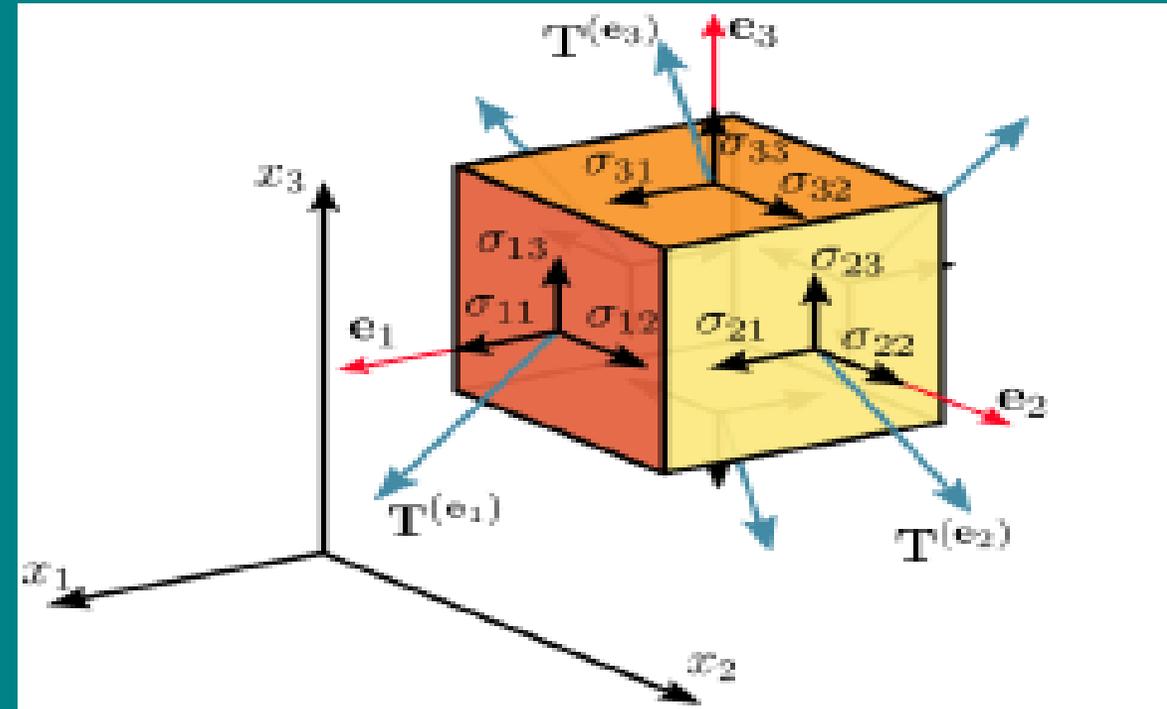
- Unit of stress?

# Stress

- $Stress = \frac{Force}{Area} = \frac{F}{A}$
- Unit of stress is Pa (Pascal) or MPa for reservoir geomechanics
- Unit of Force is N (Newton)
- Unit of Area is  $m^2$
- $1 Pa = 1 N/m^2$

# Stress tensor

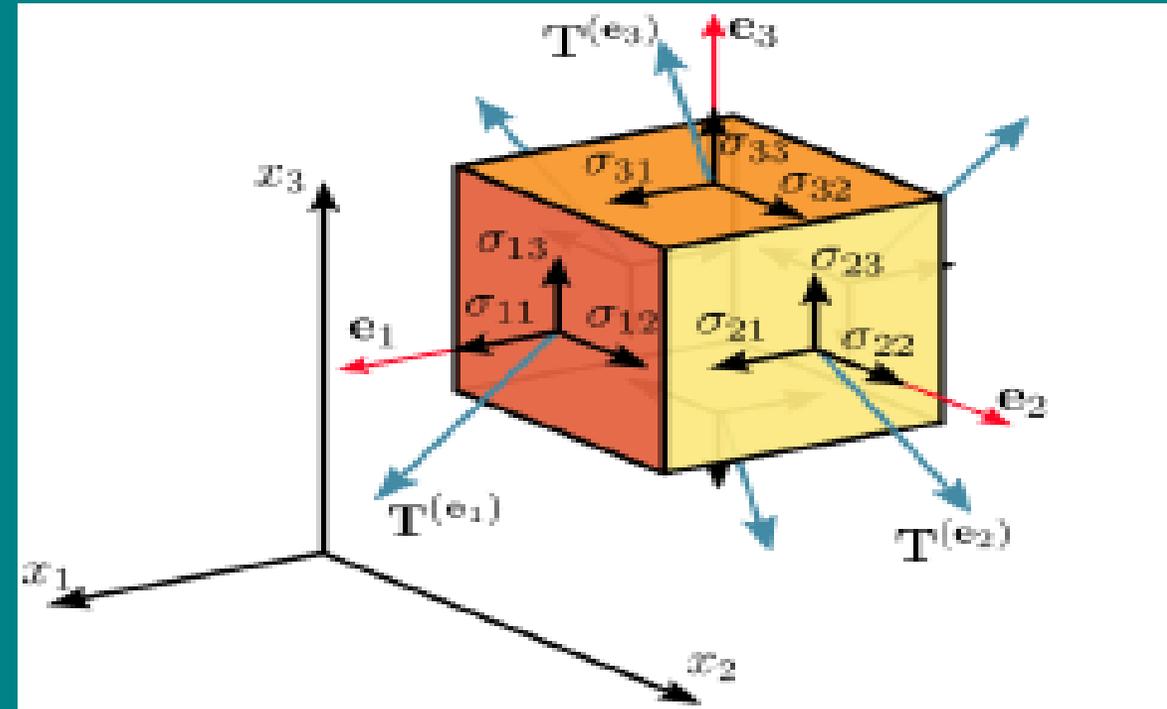
$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$
$$= \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$



Stress component: Force acting in a specific direction acting on an area of a certain orientation

# Stress tensor

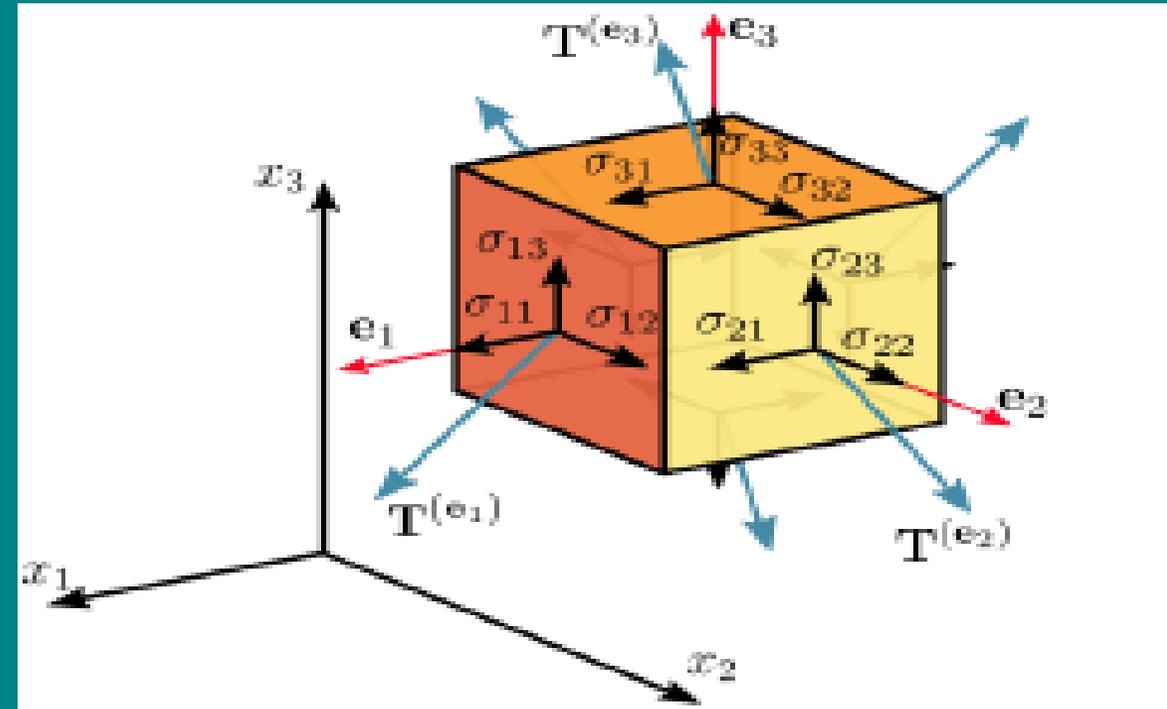
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$$= \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$



Six individual components of the stress tensor

# Stress tensor

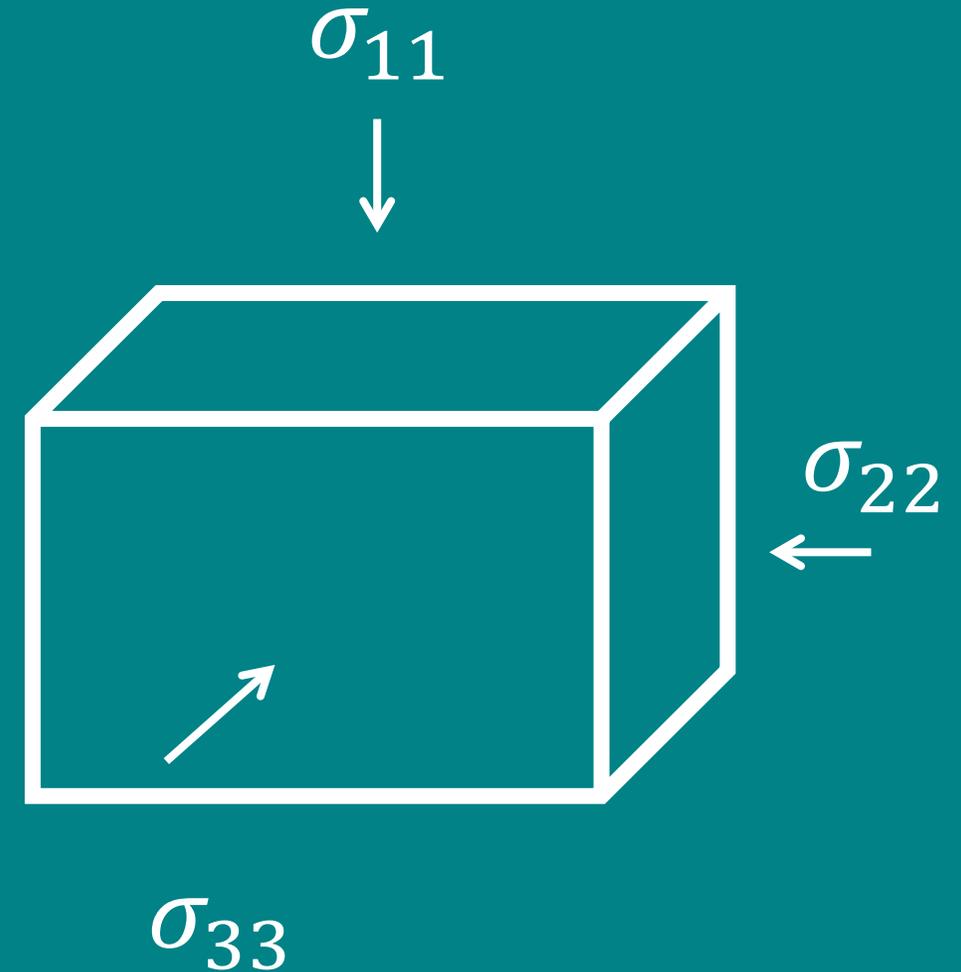
$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$
$$= \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$



Shear stress components

# Stress tensor

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$
$$= \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

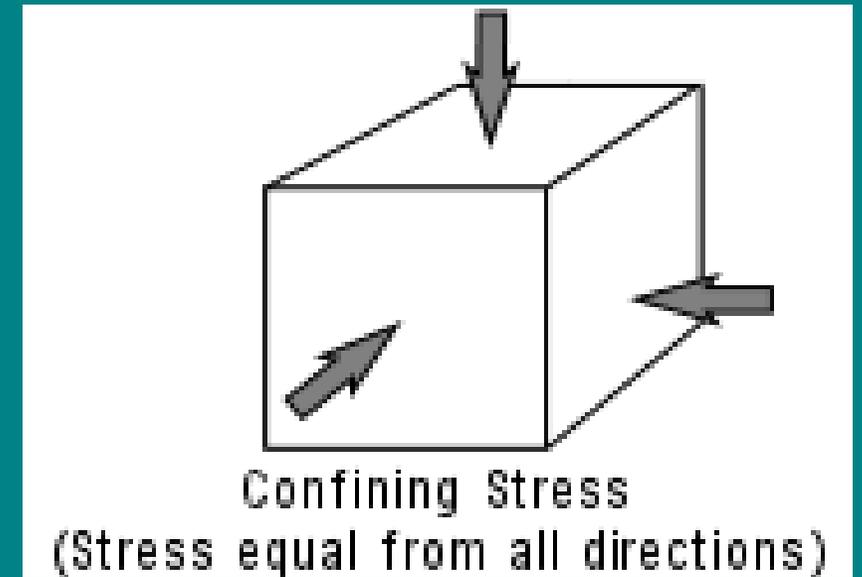


Principal / normal stress components

# Uniform normal stress

The components on the diagonal of the stress tensor,  $\sigma_{xx}$ ,  $\sigma_{yy}$ ,  $\sigma_{zz}$ , are the normal stresses and cause compression and tension of the material, since the forces are parallel with the normal of the plane on which they work.

Uniform normal stress ( $\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$ ) in an isotropic material causes a change in volume, not in shape.

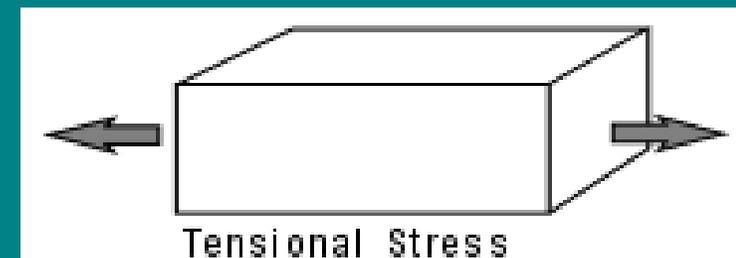
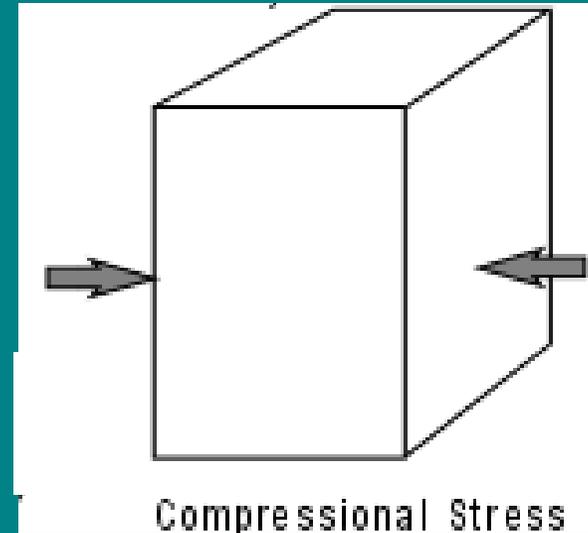


# Uniform uniaxial stress

Non-uniform normal stress ( $\sigma_{xx} \neq \sigma_{yy} \neq \sigma_{zz}$ ) cause a change in volume and shape also in isotropic materials.

Normal forces directed toward each other on the same plane (compressive stresses) cause uniaxial compression.

Normal forces in opposite direction on the same plane (tensional stresses) cause uniaxial tension.

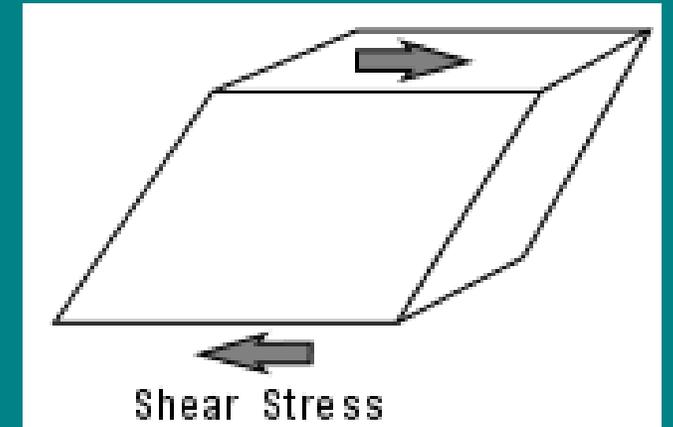


# Shear stress

The components that are not on the diagonal of the stress tensor,  $\sigma_{xy}$ ,  $\sigma_{zx}$ ,  $\sigma_{yz}$ , etc., are shear stresses and cause shear or couple in the material. The forces are perpendicular to the normal of the plane on which they work.

For pure shear the forces are parallel but in opposite directions on the opposite planes.

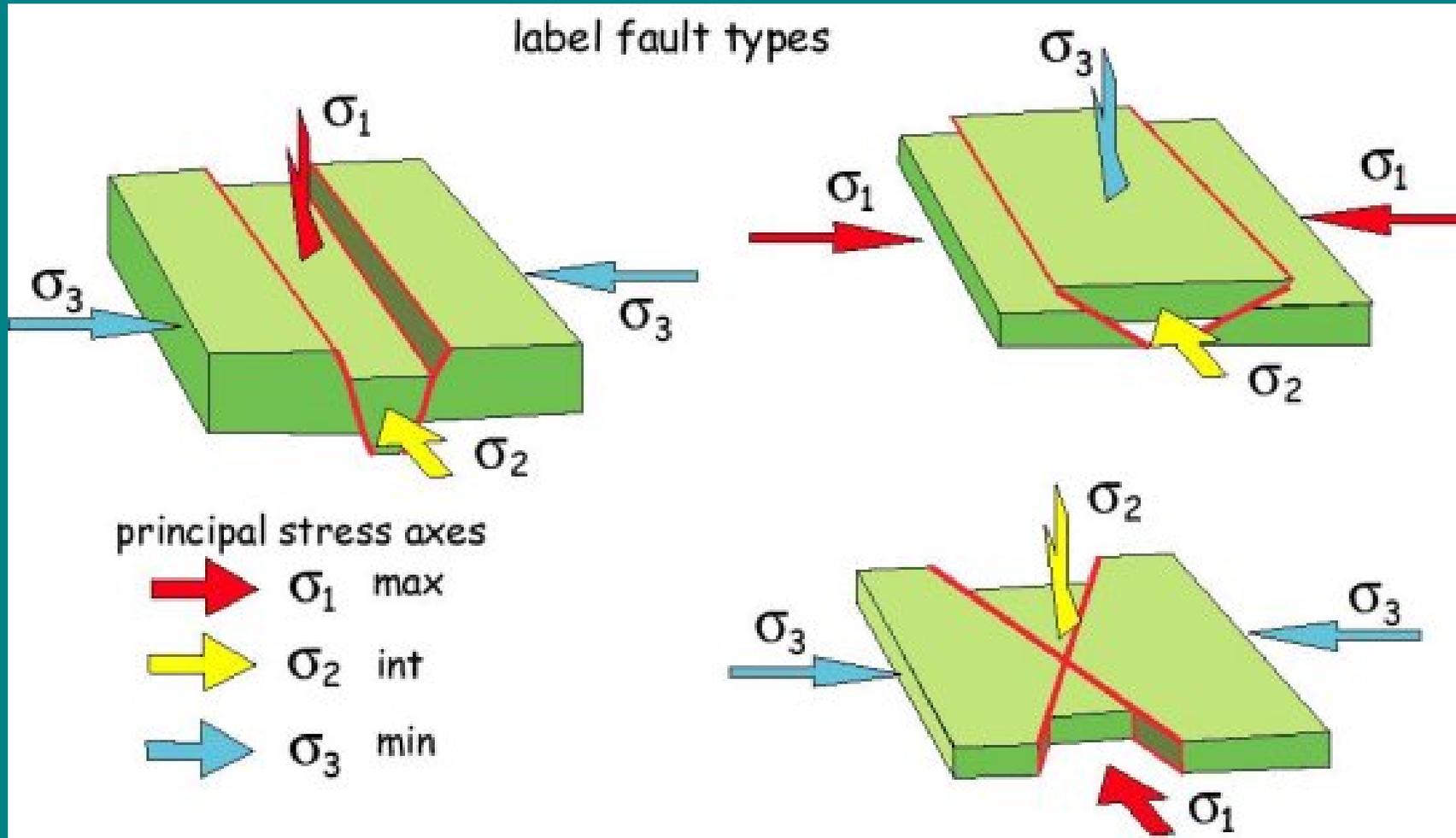
An isotropic material changes shape without changing volume.



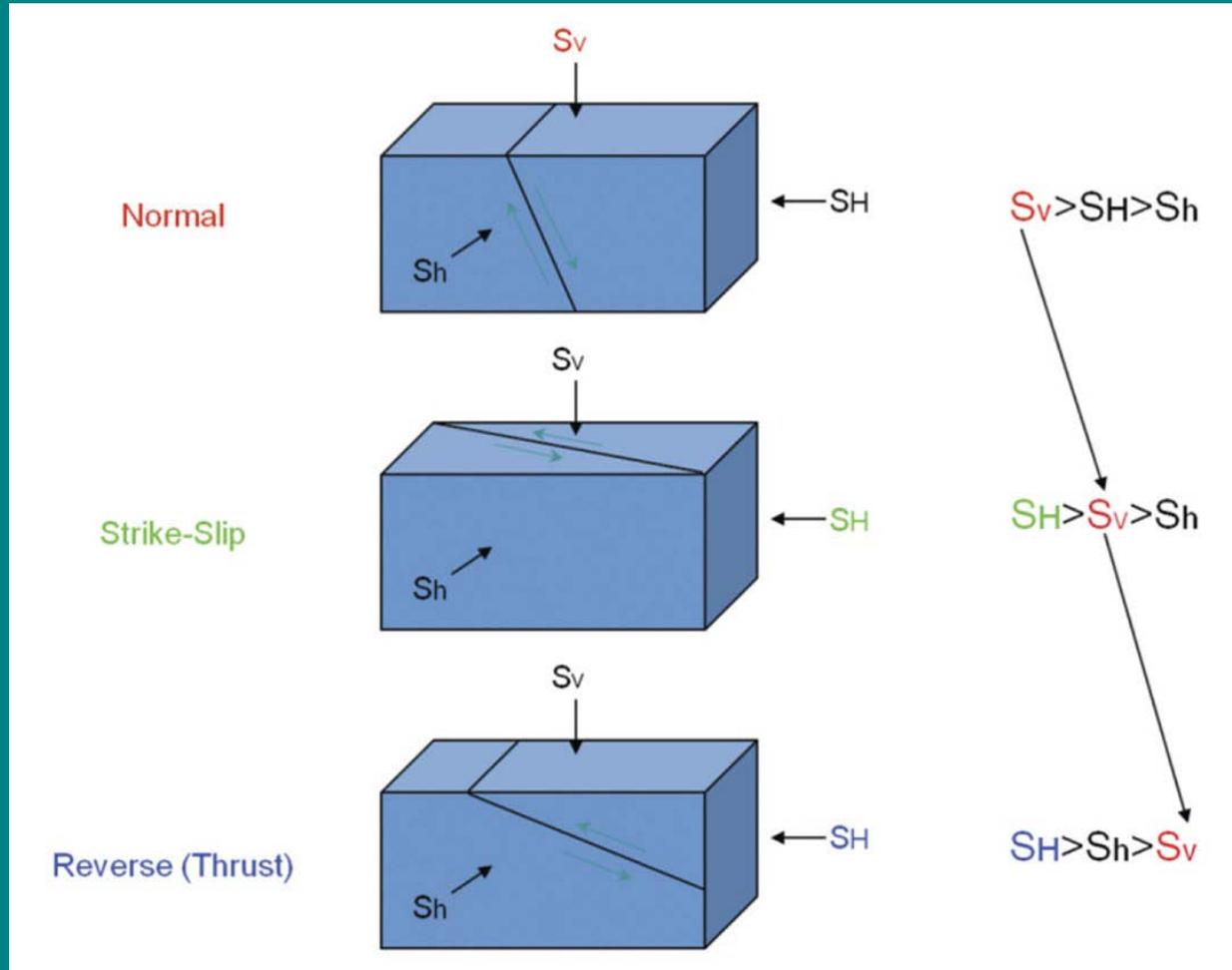
# Stress conditions in the Earth

- Difference in magnitude between the three principal stress conditions controls favourable orientation of growth of faults and fractures
- Direction of the 3 principal stresses controls fracture growth direction in an Earth reference frame AND have control current fracture opening.

# Stress conditions in the Earth

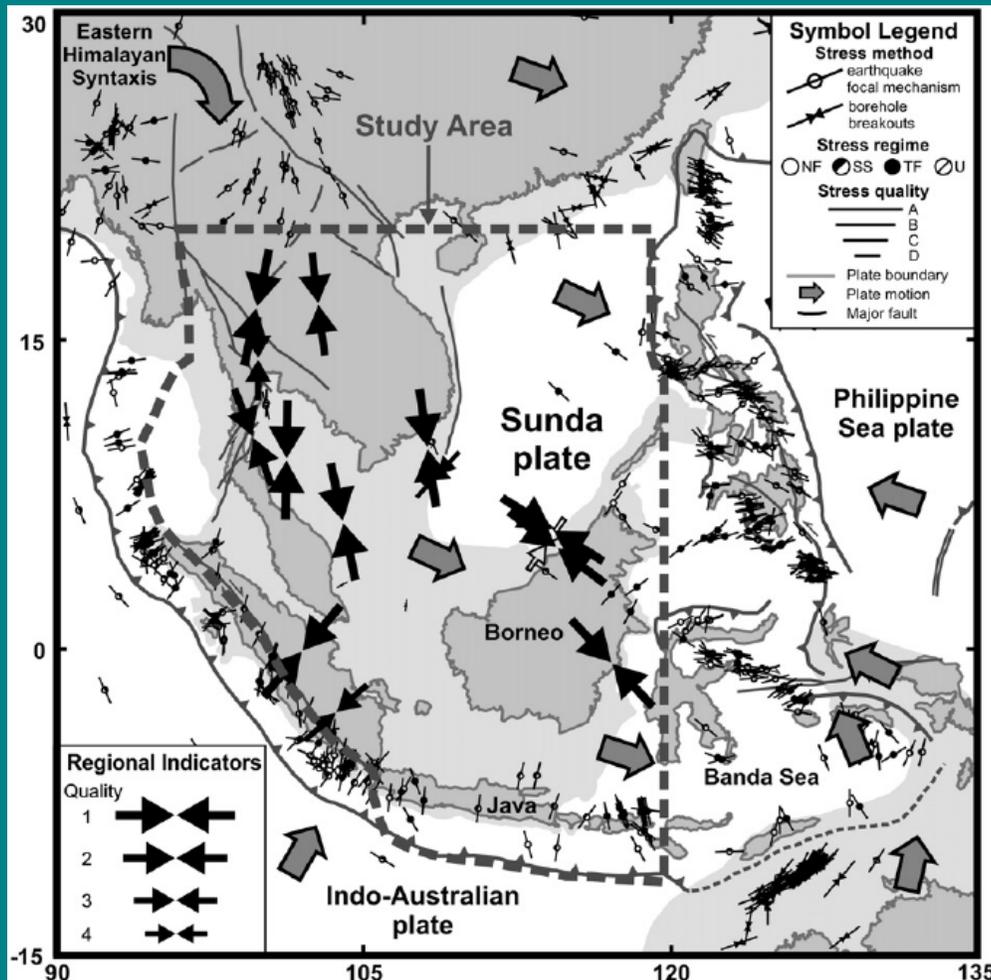


# Stress conditions in the Earth



<http://csegrecorder.com/articles/view/geomechanics-bridging-the-gap-from-geophysics-to-engineering>

# Horizontal stresses in Indonesia

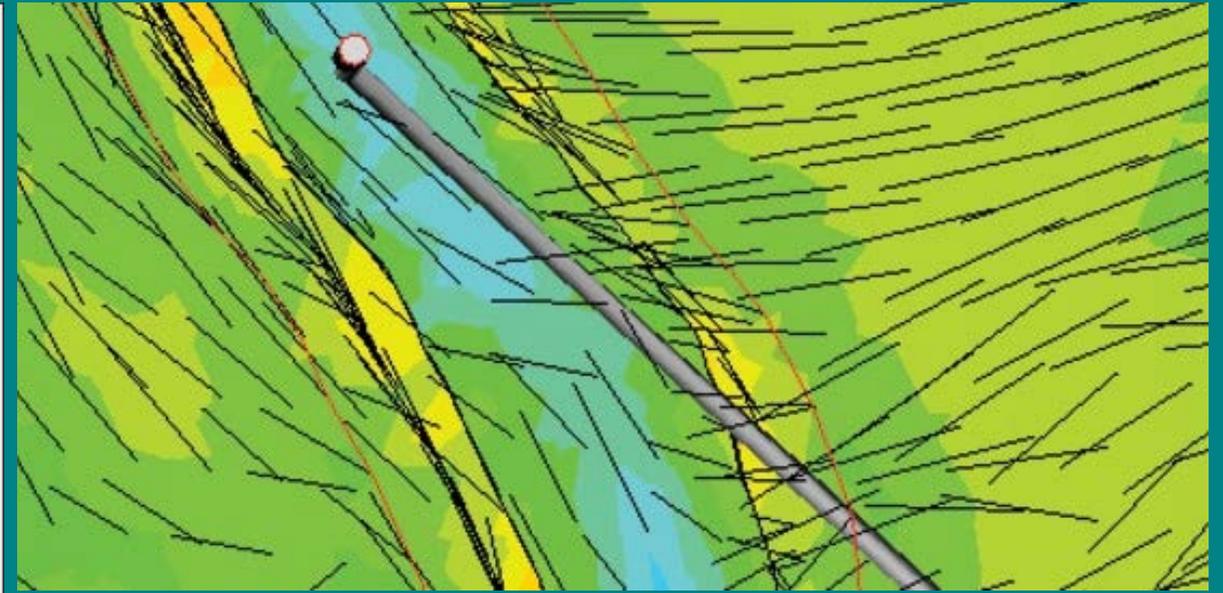
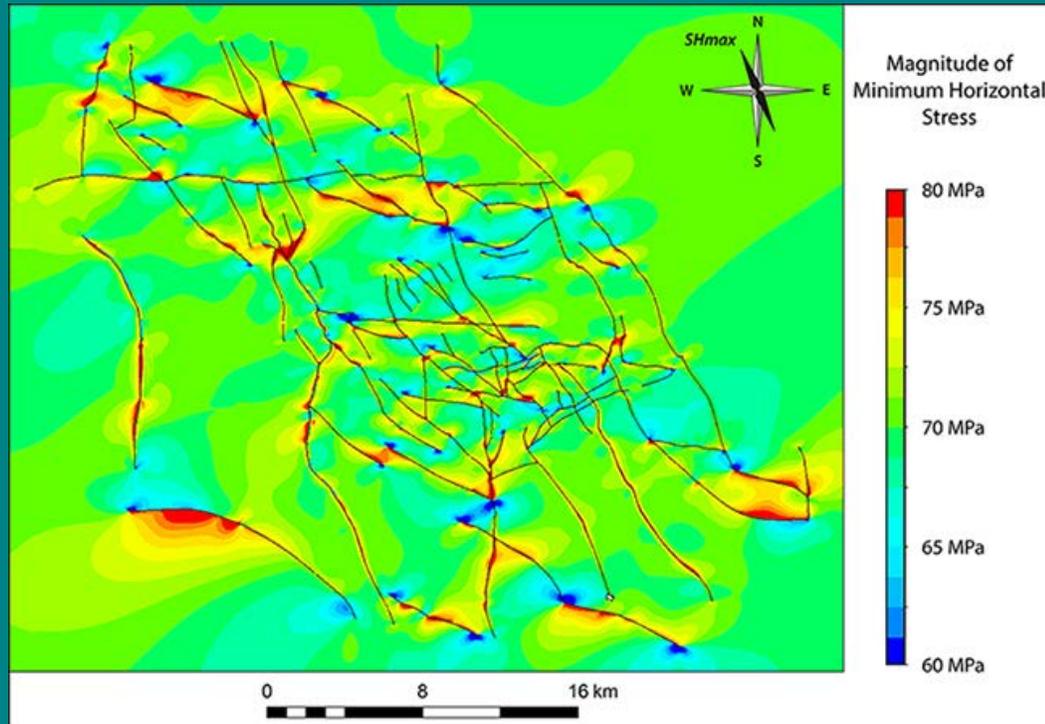


Tingay et al.:

'NE–SW in Sumatra, and NNE–SSW in Java, with the NE–SW stress direction in Sumatra confirmed from borehole breakouts and hydraulic fracture tests (Fig. 7, Mount and Suppe, 1992)'

Tingay et al., 2010.

# Local stress magnitude and direction



[http://www.golder.com/ph/modules.php?name=Newsletters&op=viewarticle&sp\\_id=167&page\\_id=1100&article\\_id=570](http://www.golder.com/ph/modules.php?name=Newsletters&op=viewarticle&sp_id=167&page_id=1100&article_id=570)

[http://www.geo.tu-darmstadt.de/fg/inggeol/inggeo\\_forschung/dgmk\\_forschungsprojekt\\_721/dgmk\\_project\\_721.en.jsp](http://www.geo.tu-darmstadt.de/fg/inggeol/inggeo_forschung/dgmk_forschungsprojekt_721/dgmk_project_721.en.jsp)

Local stresses can vary significantly from tectonic stresses

# Strain

- Deformation is the transformation of a body from a *reference* configuration to a *new* configuration
- Strain is a description of deformation in terms of *relative* displacement of particles.

# Strain



# Strain

- Strain is a description of deformation in terms of *relative* displacement of particles.
- Strain is the length change of a body with respect to its original length

$$\varepsilon = \frac{\Delta L}{L}$$

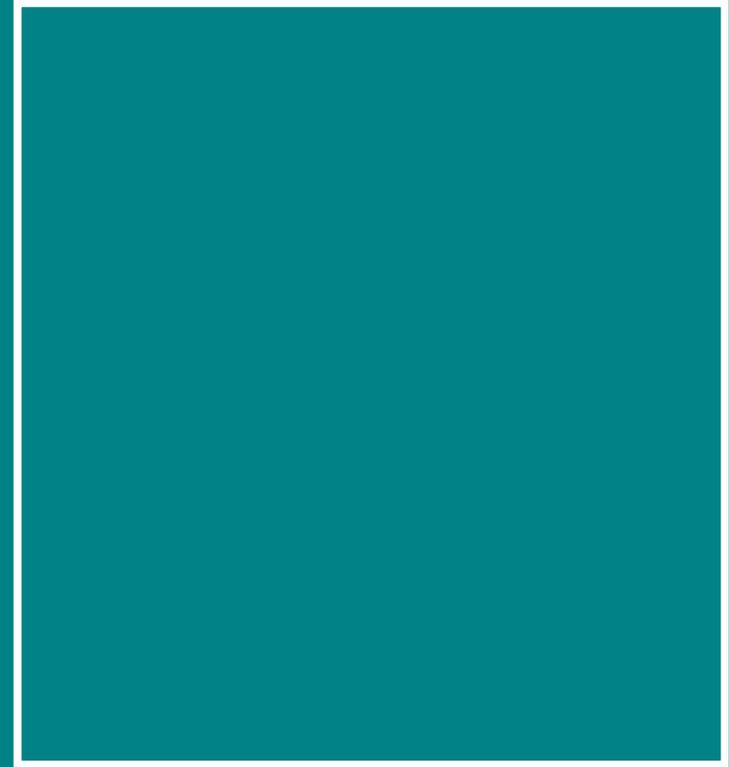
$$\varepsilon = \frac{L_{new} - L}{L}$$

- Unit of strain?

# Strain

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{L_{new} - L}{L}$$

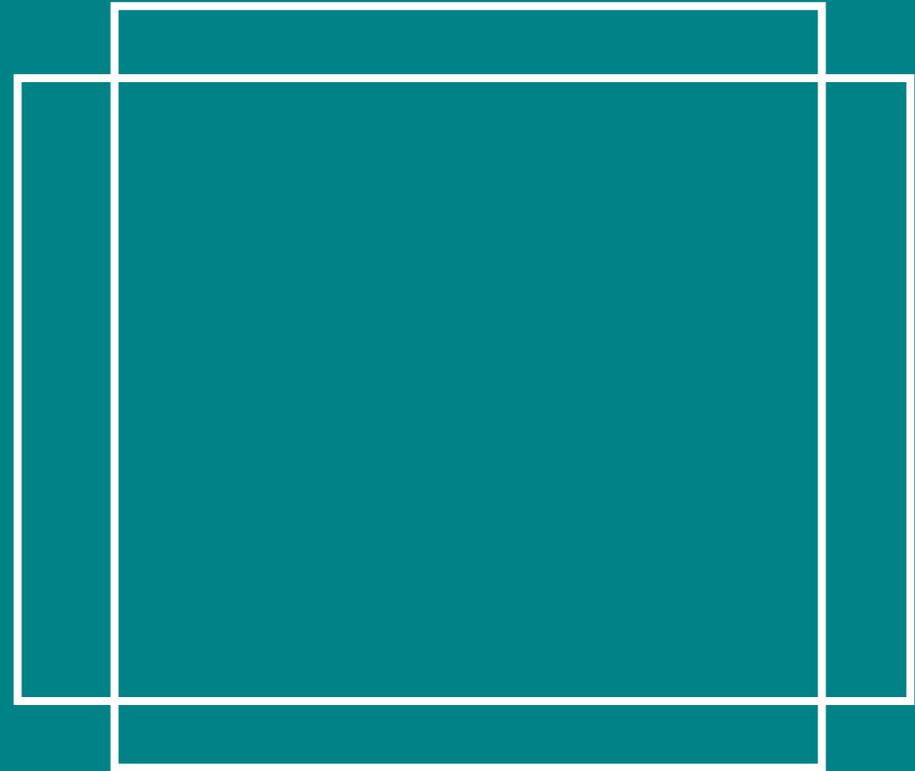


# Strain

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{L_{new} - L}{L}$$

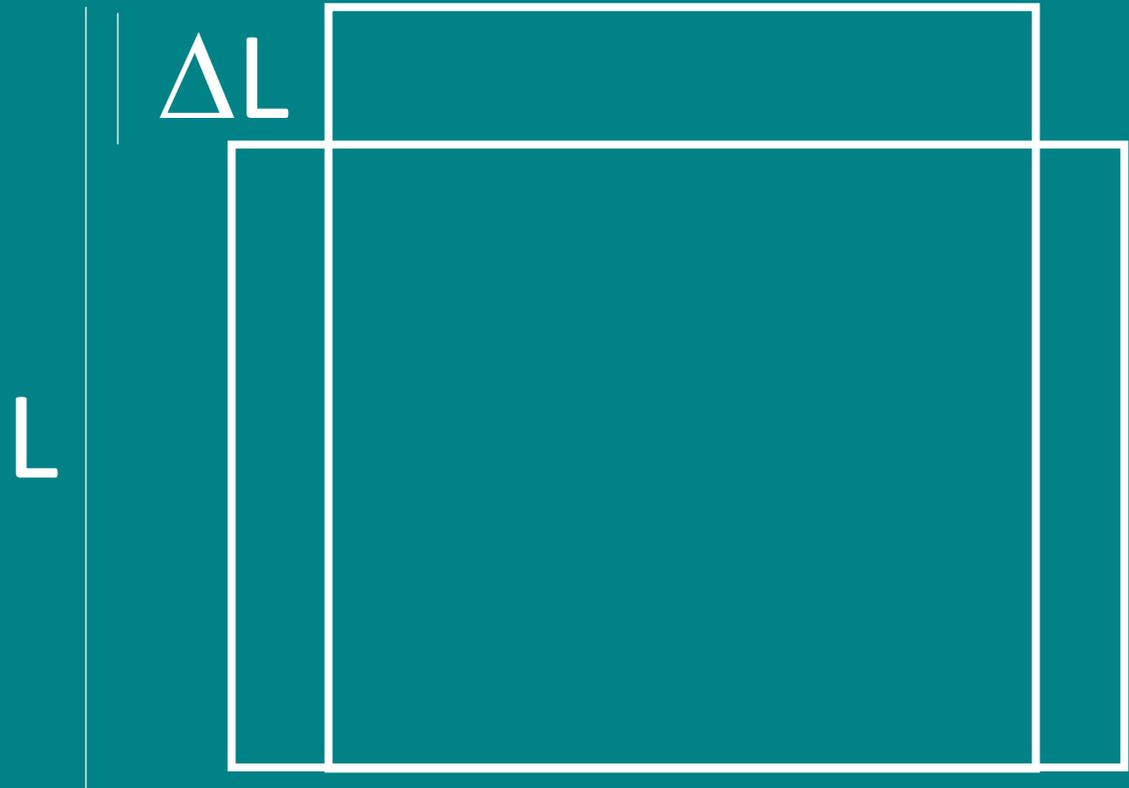
L



# Strain

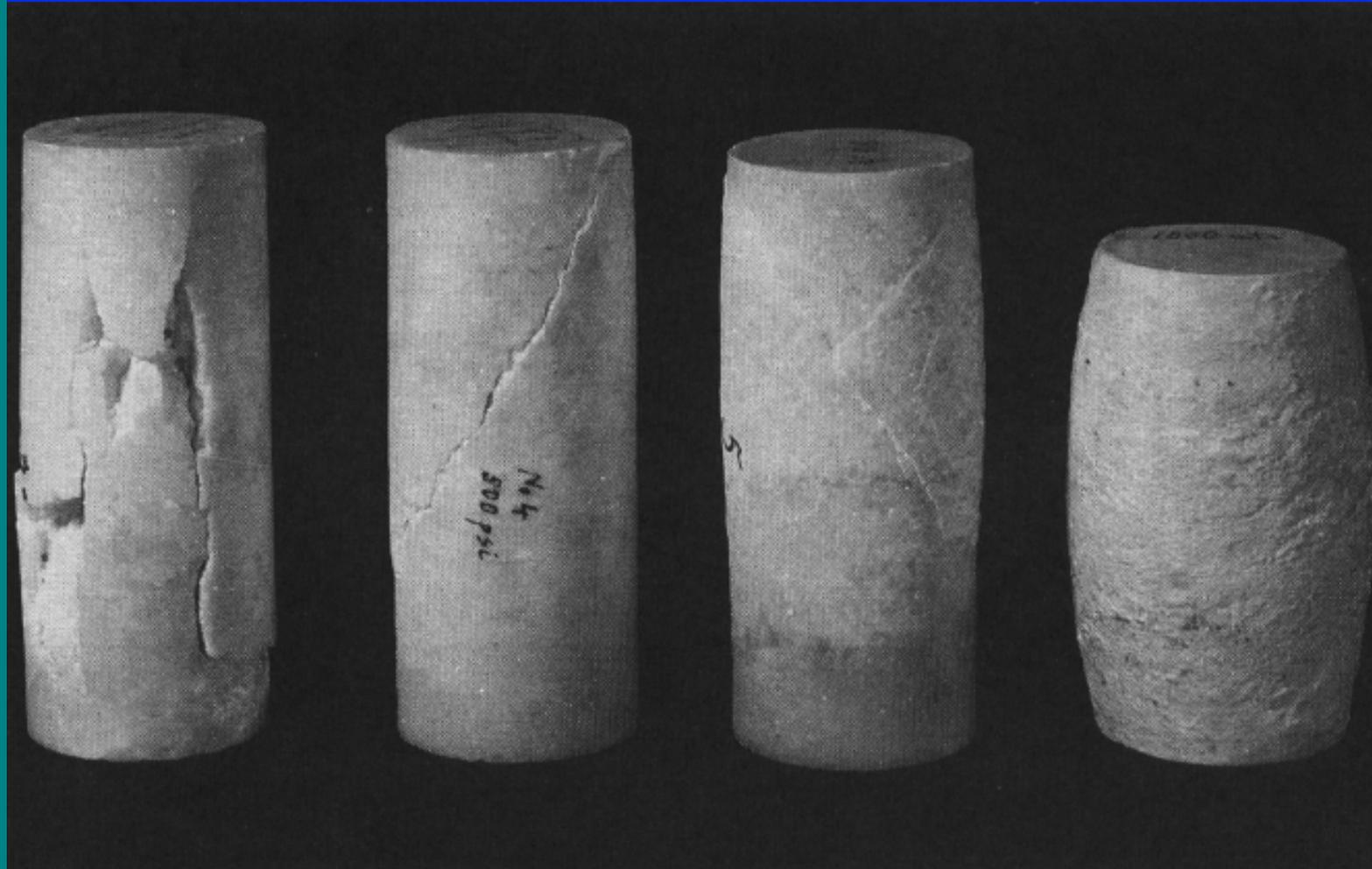
$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{L_{new} - L}{L}$$



Assumption: No volume change of the body

# Strain in lab experiments

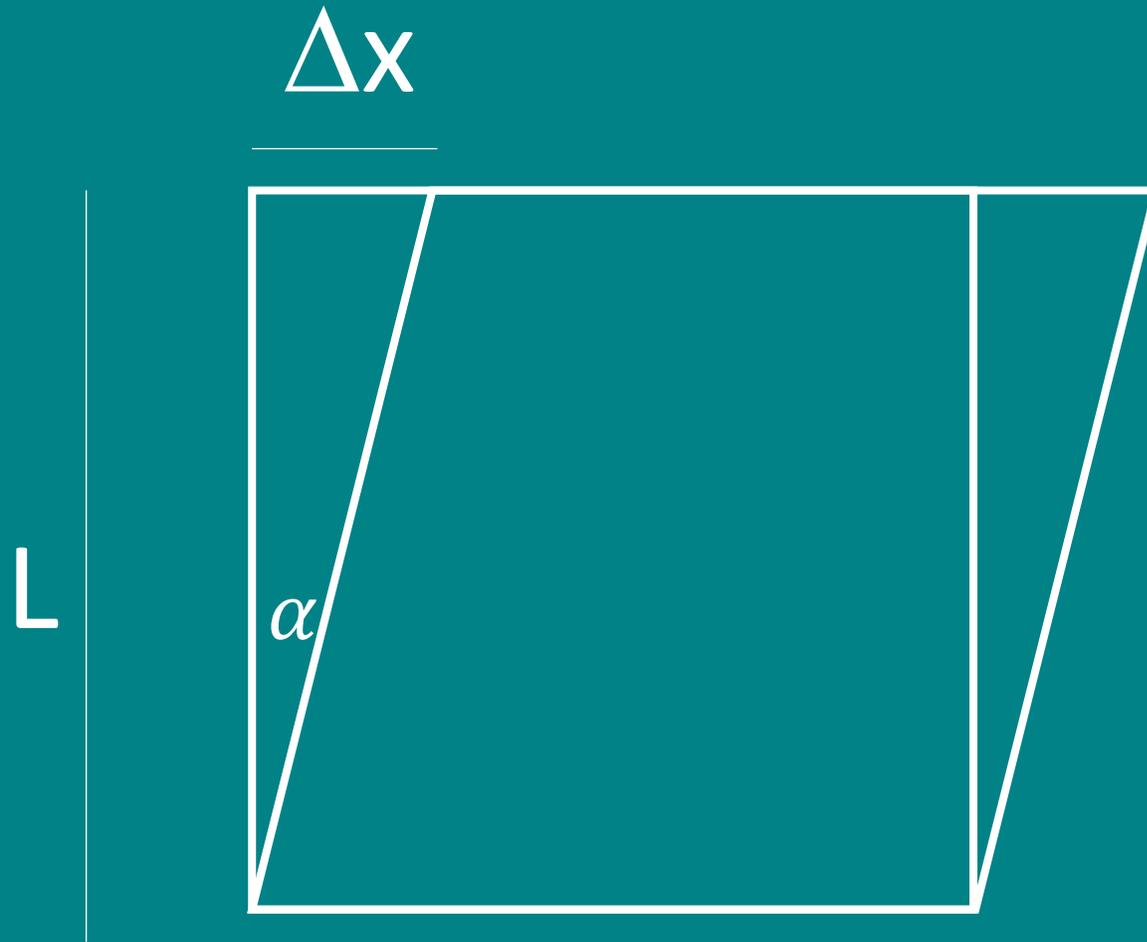


[geode.colorado.edu](http://geode.colorado.edu)

# Shear strain



# Shear strain



$$\gamma = \frac{\Delta x}{L} = \tan \alpha$$

$\Delta x$  = Displacement / Deformation

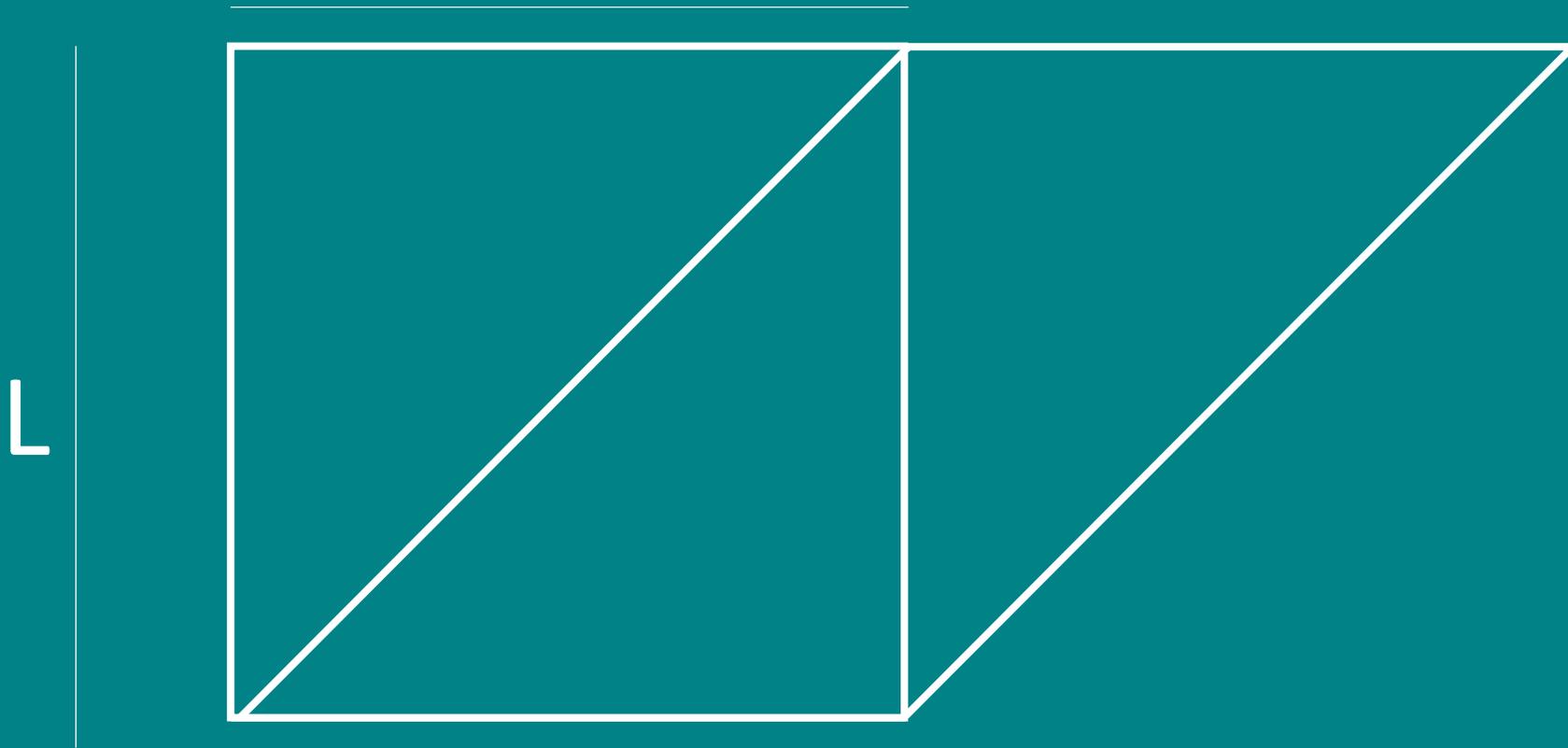
$L$  = Original Length

$\gamma$  = Shear strain

# Shear strain

$$\Delta x = L$$

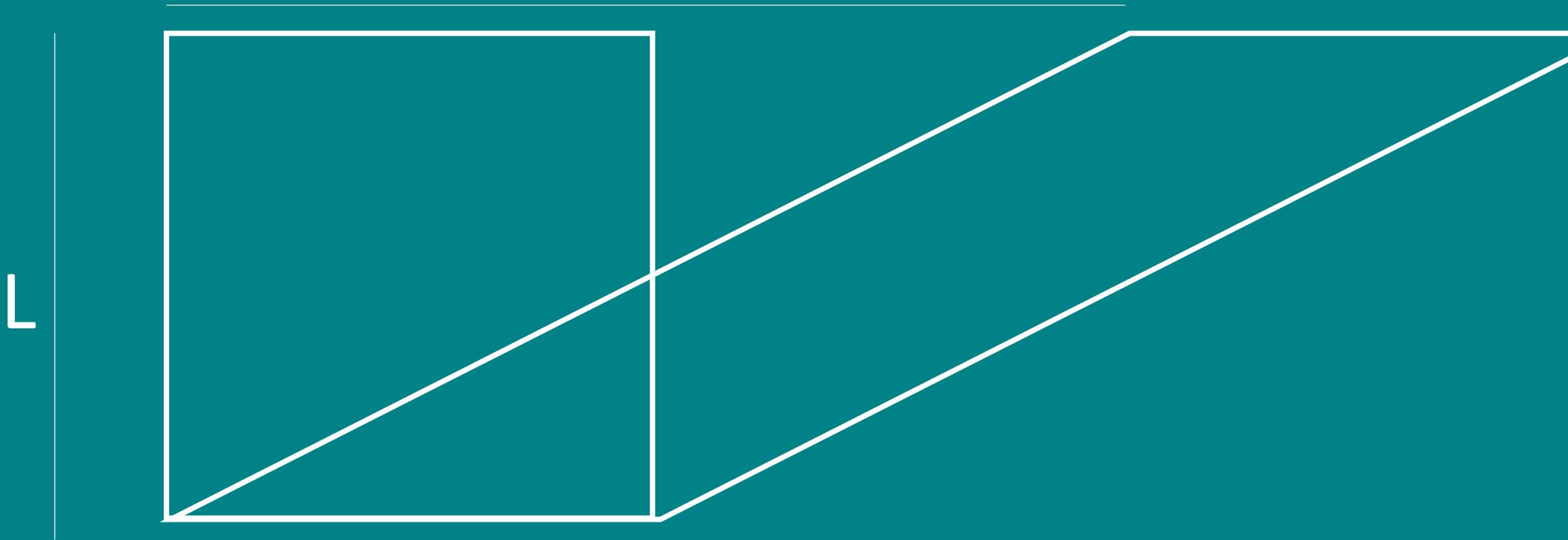
$$\gamma = 1$$



# Shear strain

$$\Delta x = 2L$$

$$\gamma = 2$$



# In-class stress-strain exercise - 1

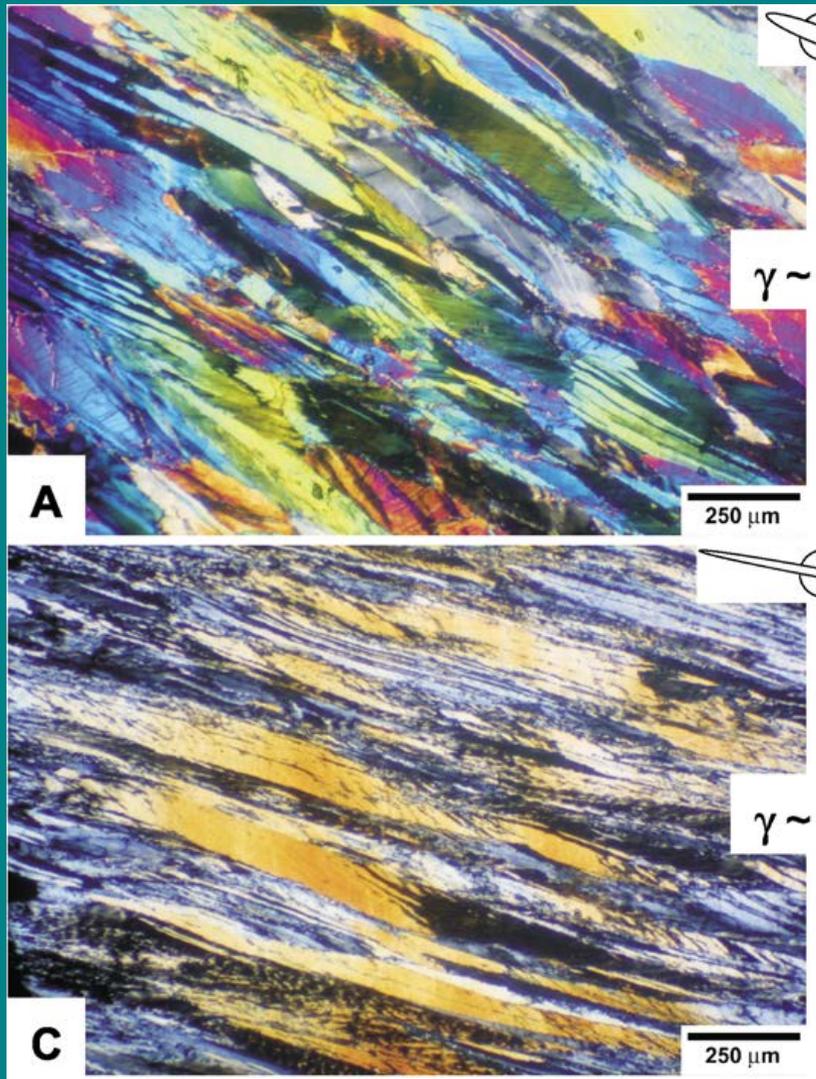
## 10 min



Lavallee et al., 2008

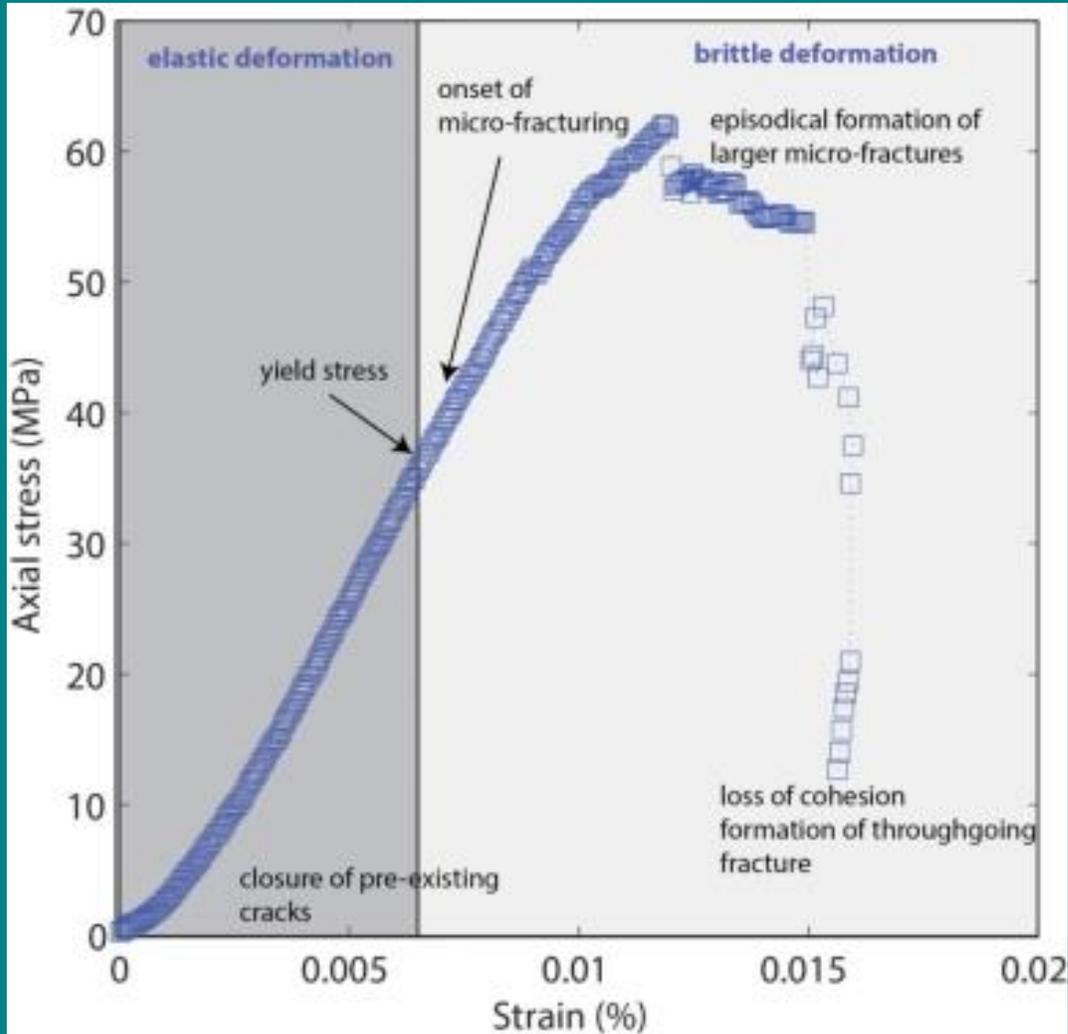
- Calculate the vertical and horizontal strain for each of these samples. The sample at the left is the original undeformed sample.

# In-class stress-strain exercise - 2



- What is the shear strain of these 2 samples? The lineations in the images indicate the long axes of the parallelogram.

# In-class stress-strain exercise - 3

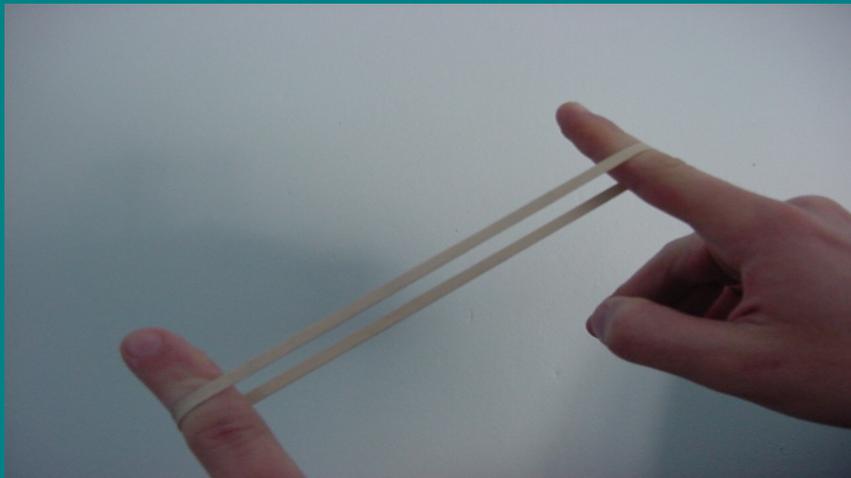


- What is the stress-strain ratio ( $\Delta\sigma/\Delta\varepsilon$ ) of this experimentally deformed shale sample? Determine the stress-strain ratio only in the gray part of the diagram. The unit of the stress-strain ratio is in GPa.

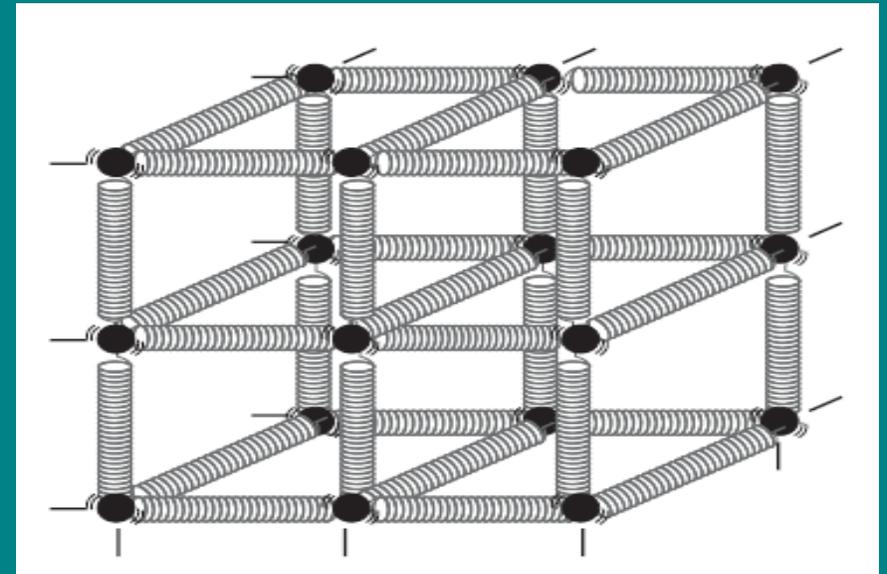
# 15 minute break

# Elastic behaviour

- Definition of elasticity
  - In physics, **elasticity** is a physical property of materials which return to their original shape after the stress that caused their deformation is no longer applied.

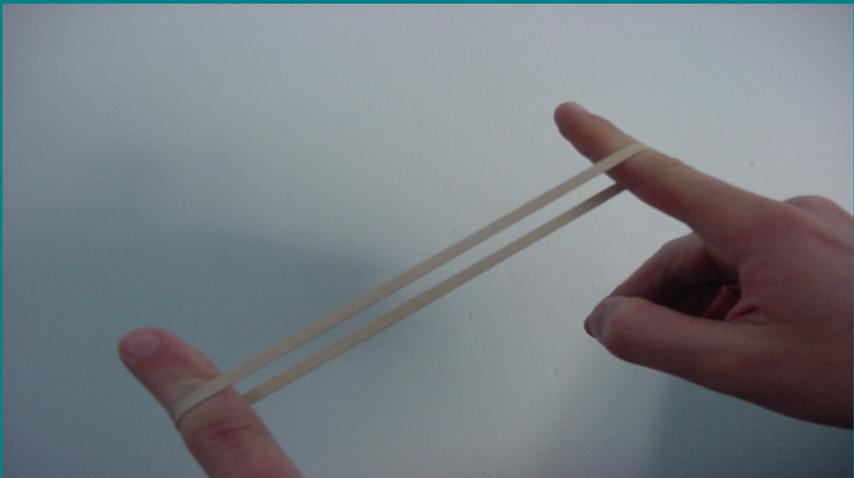


[http://en.wikipedia.org/wiki/Elasticity\\_%28physics%29](http://en.wikipedia.org/wiki/Elasticity_%28physics%29)

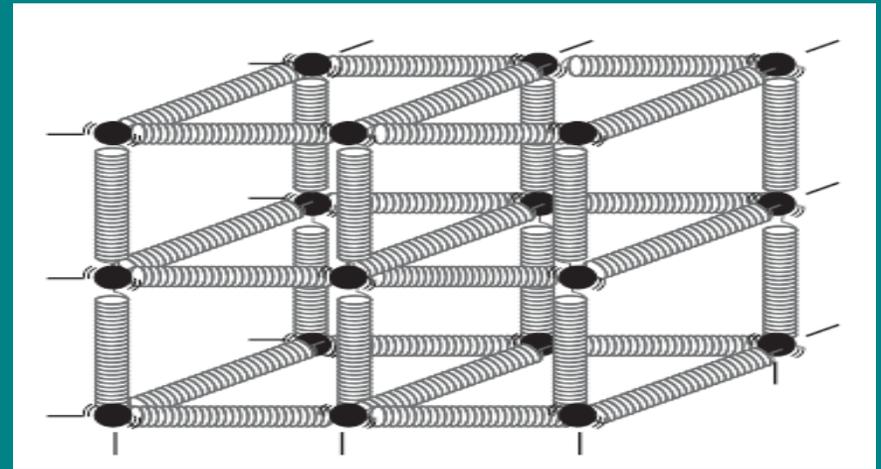


# Elastic behaviour

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[http://en.wikipedia.org/wiki/Elasticity\\_%28physics%29](http://en.wikipedia.org/wiki/Elasticity_%28physics%29)

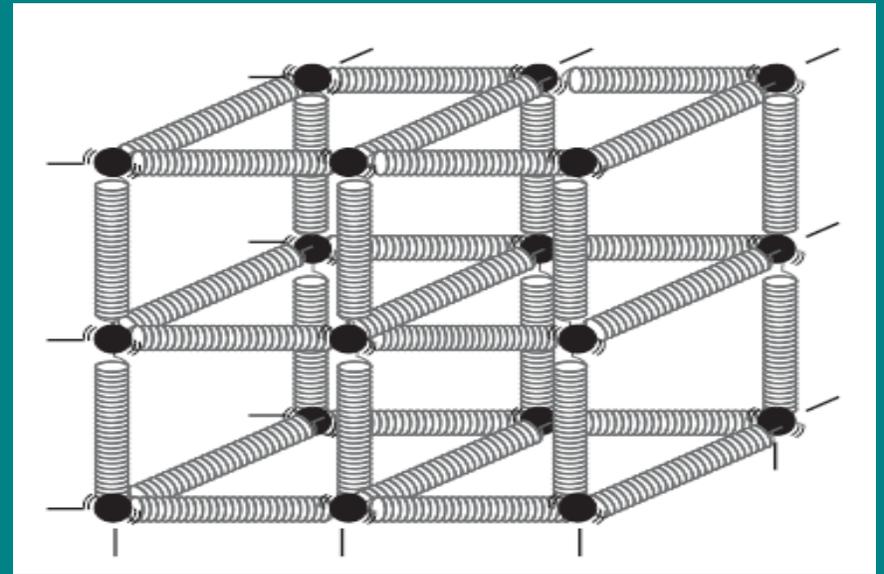


# Elastic behaviour

- Definition of elasticity
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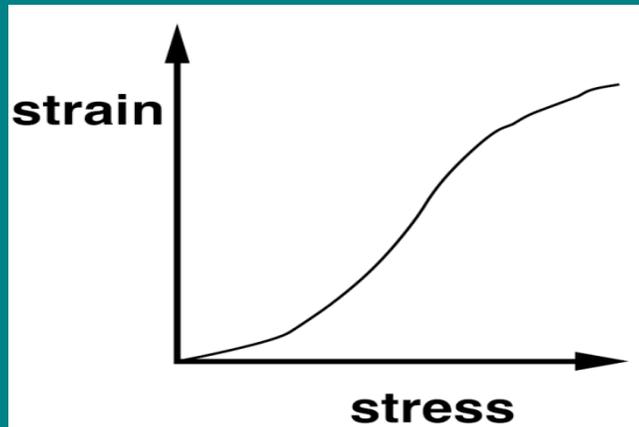
[http://en.wikipedia.org/wiki/Elasticity\\_%28physics%29](http://en.wikipedia.org/wiki/Elasticity_%28physics%29)



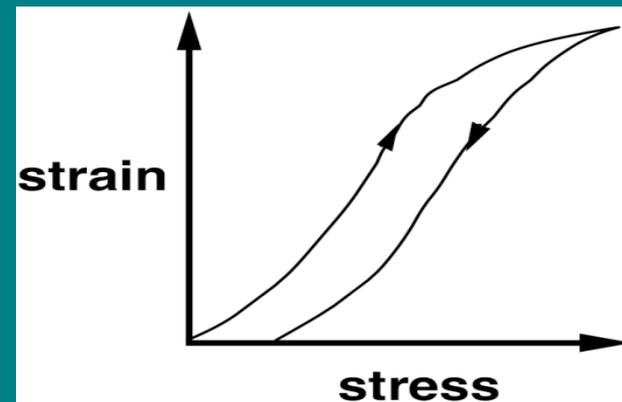
# Elastic vs. inelastic behaviour

A material is elastic if:

- The material returns to its unique relaxed state after the applied loads are removed.
- The displacement and strain are independent of the loading history (i.e. no hysteresis).



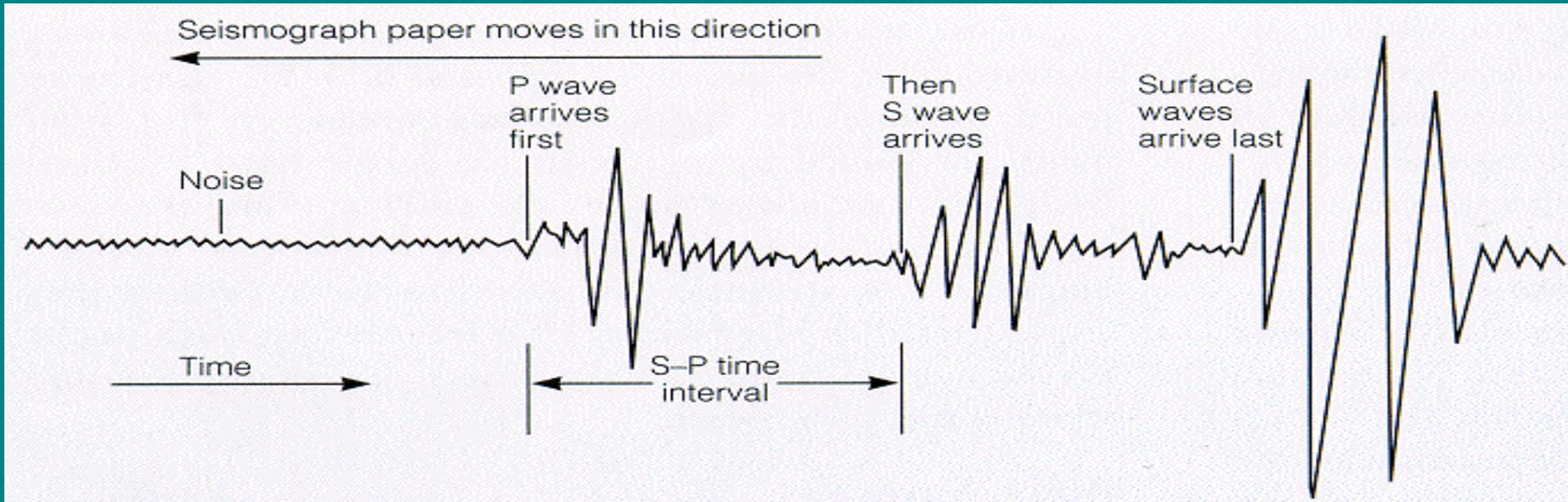
Unique stress-strain relation



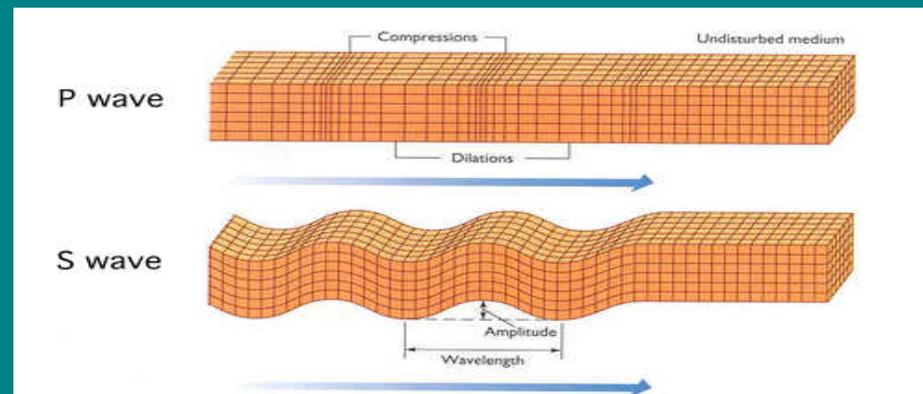
Mavko, 2003

Non-unique stress-strain relation

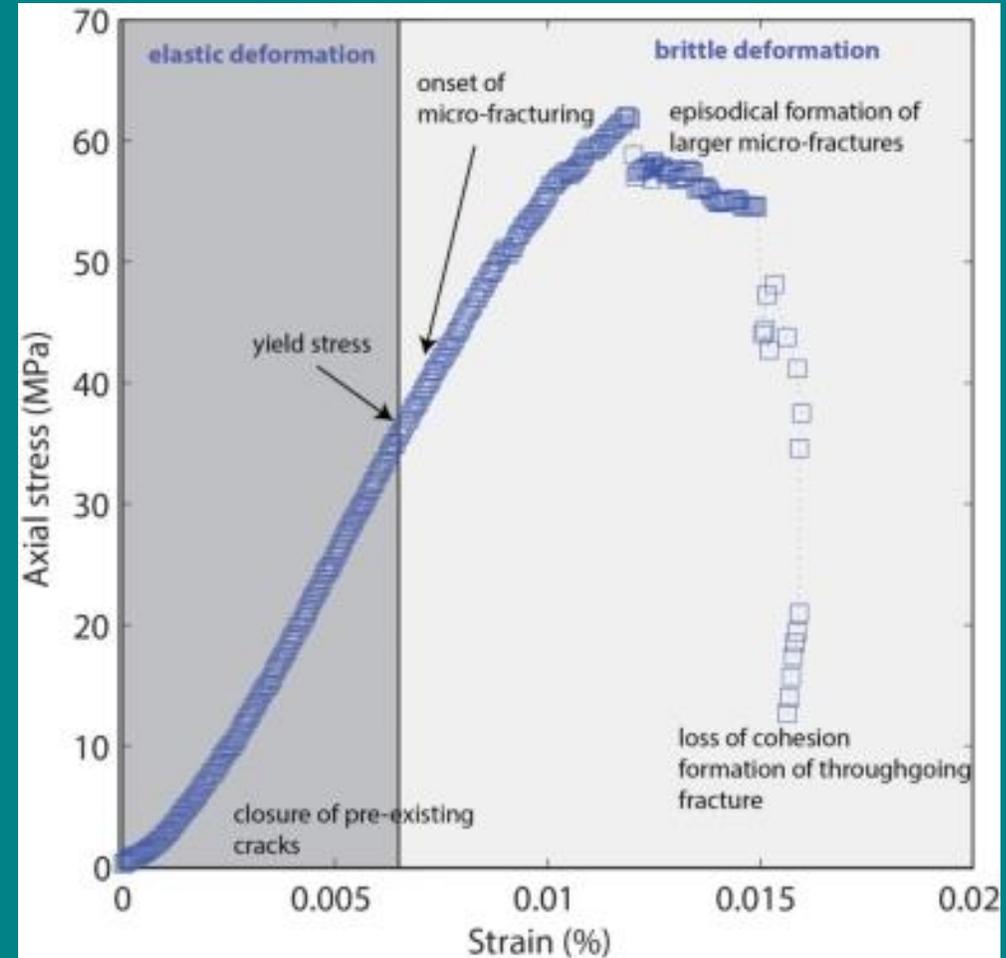
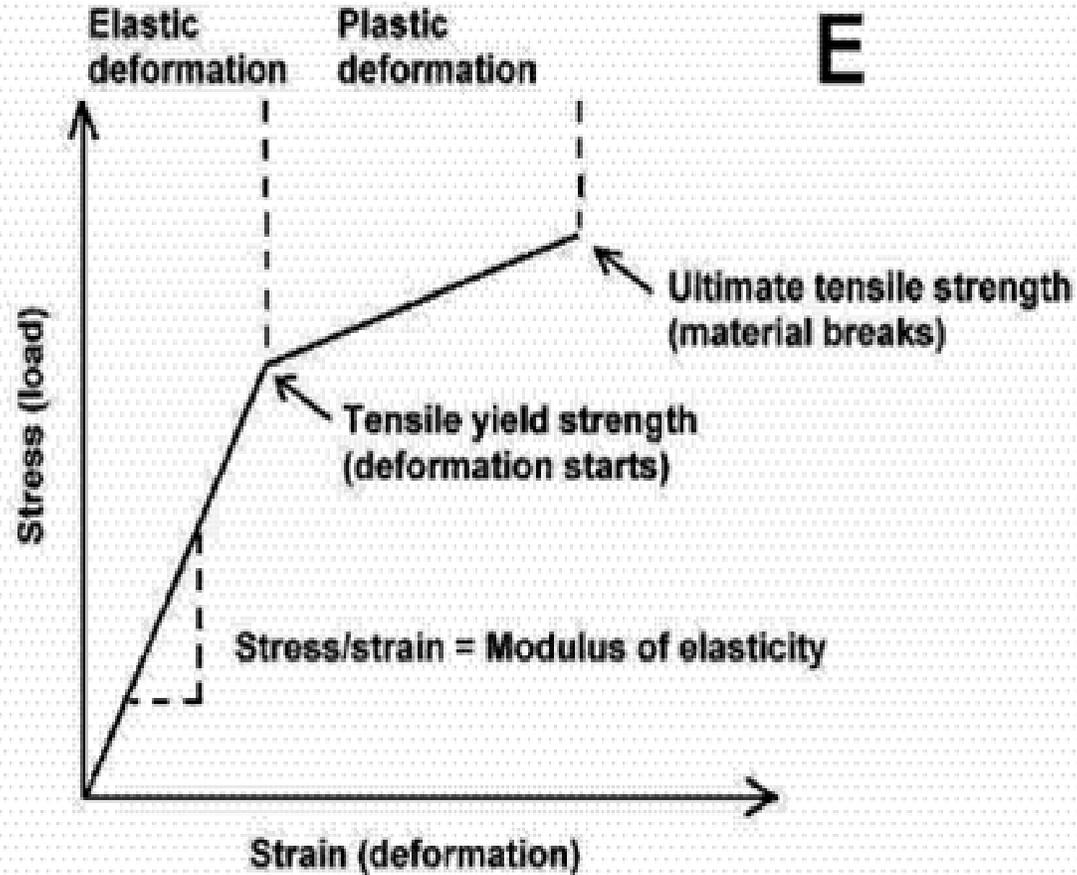
# Seismic waves



[http://www.tjhsst.edu/~jlafever/wanimate/Wave\\_Properties2.html](http://www.tjhsst.edu/~jlafever/wanimate/Wave_Properties2.html)



# Elasticity



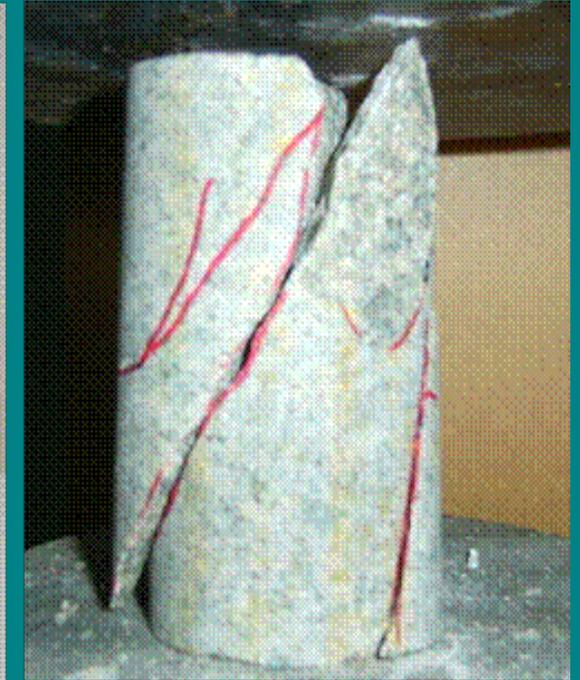
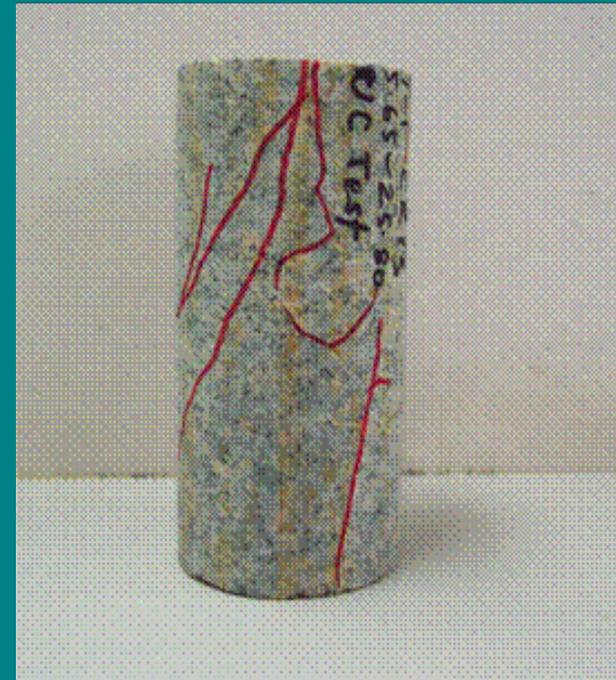
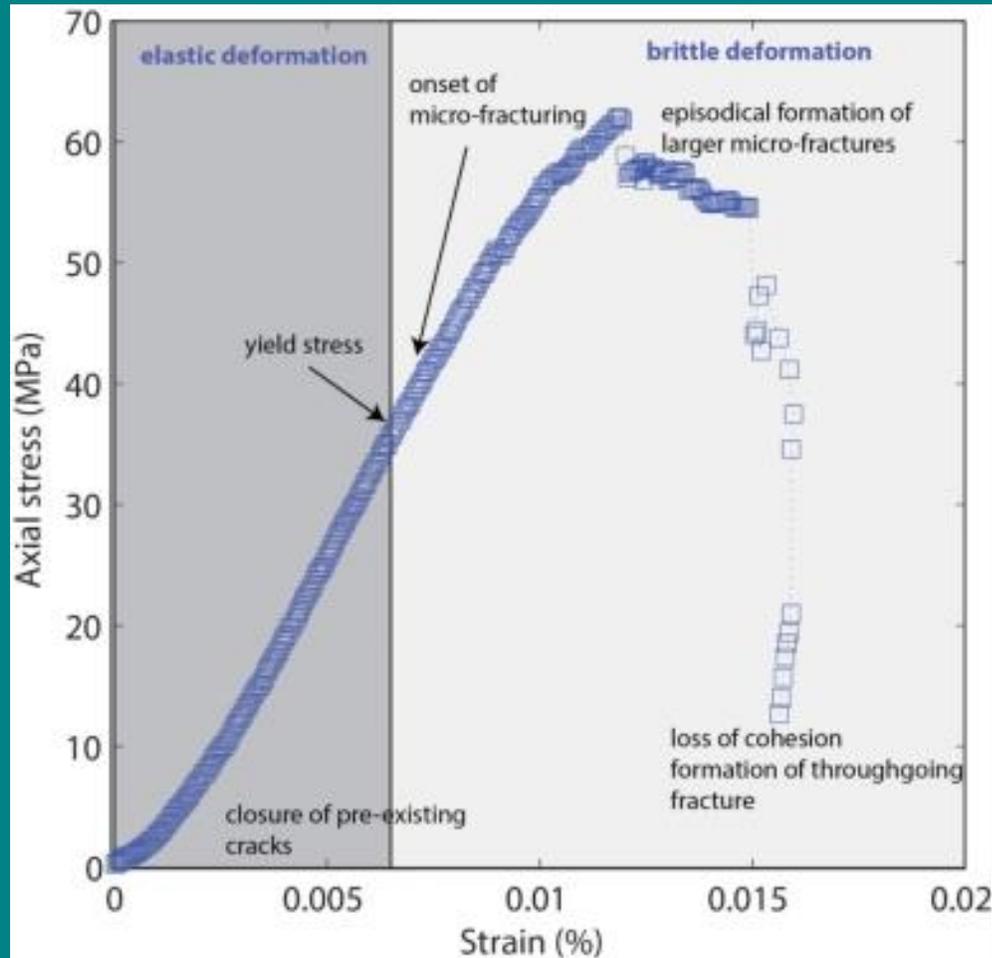
# Inelastic behaviour

## Transition to inelasticity

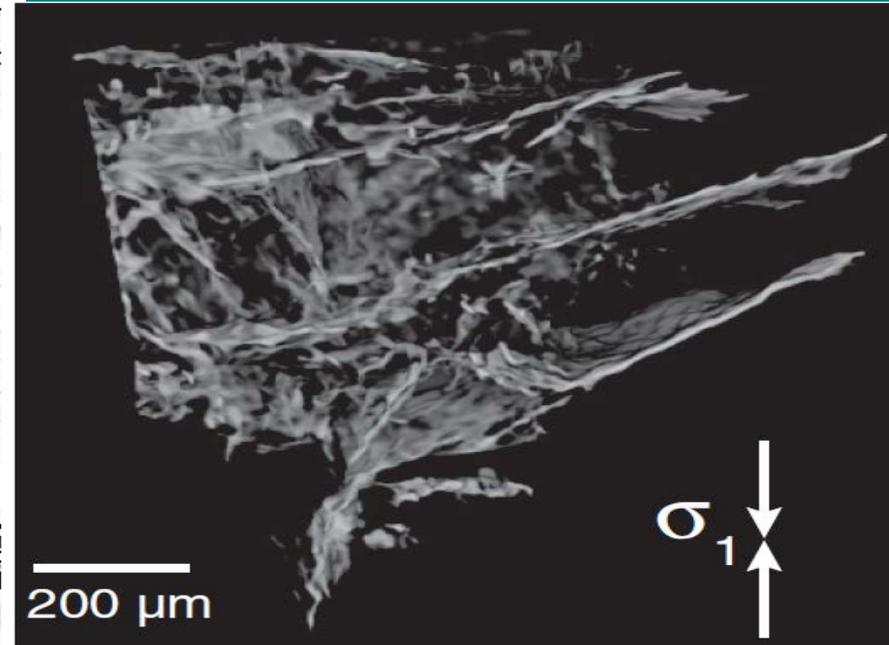
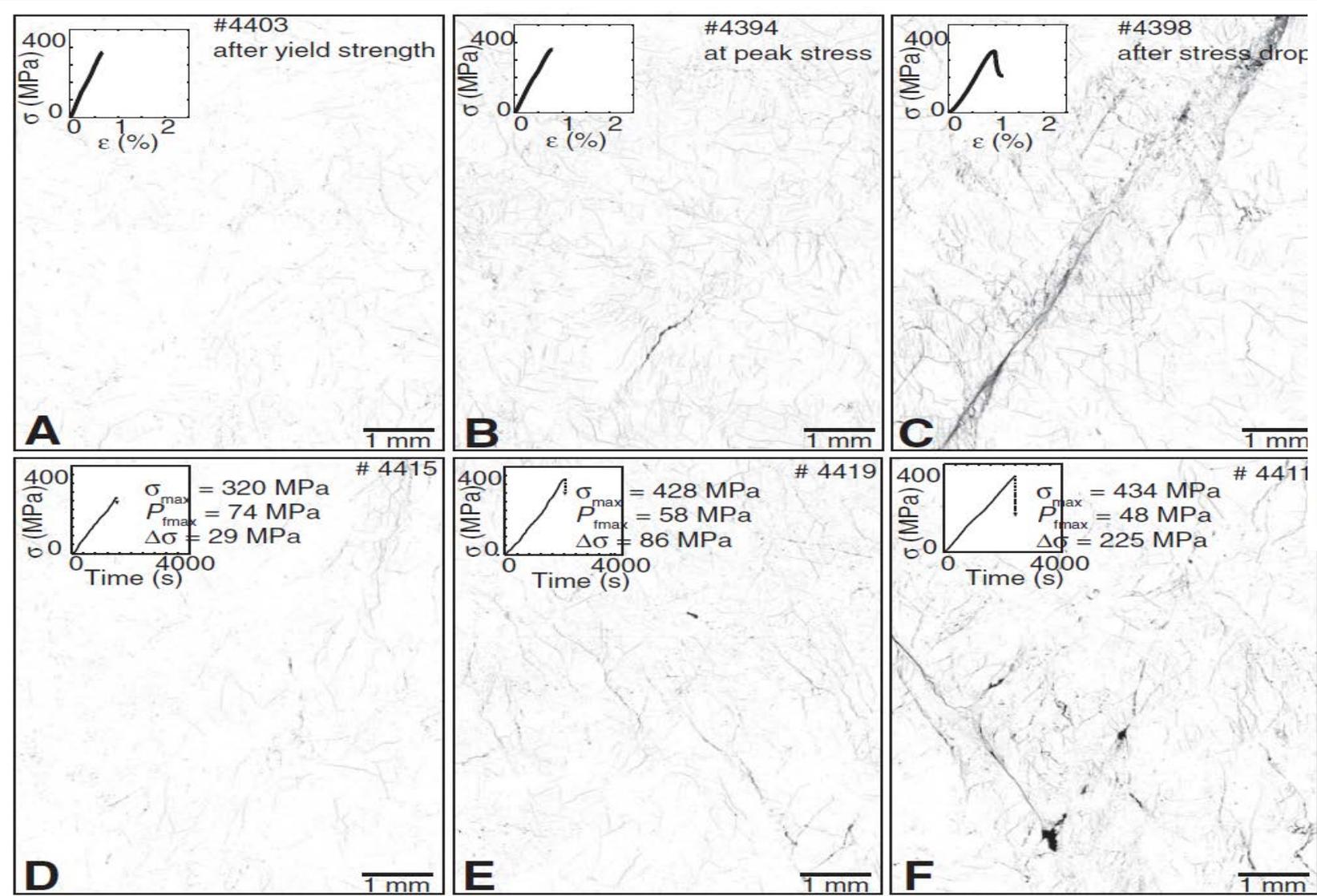
Above a certain stress known as the elastic limit or the yield strength of an elastic material, the relationship between stress and strain becomes non-linear. Beyond this limit, the solid may deform irreversibly, exhibiting plasticity.



# Brittle deformation: Typical rupture

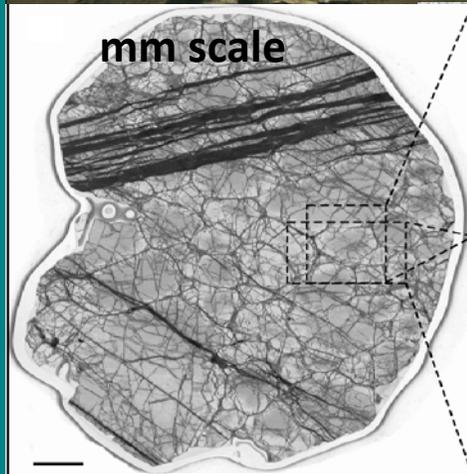


# Brittle deformation: Experiments



Very low strain before fracturing occurs

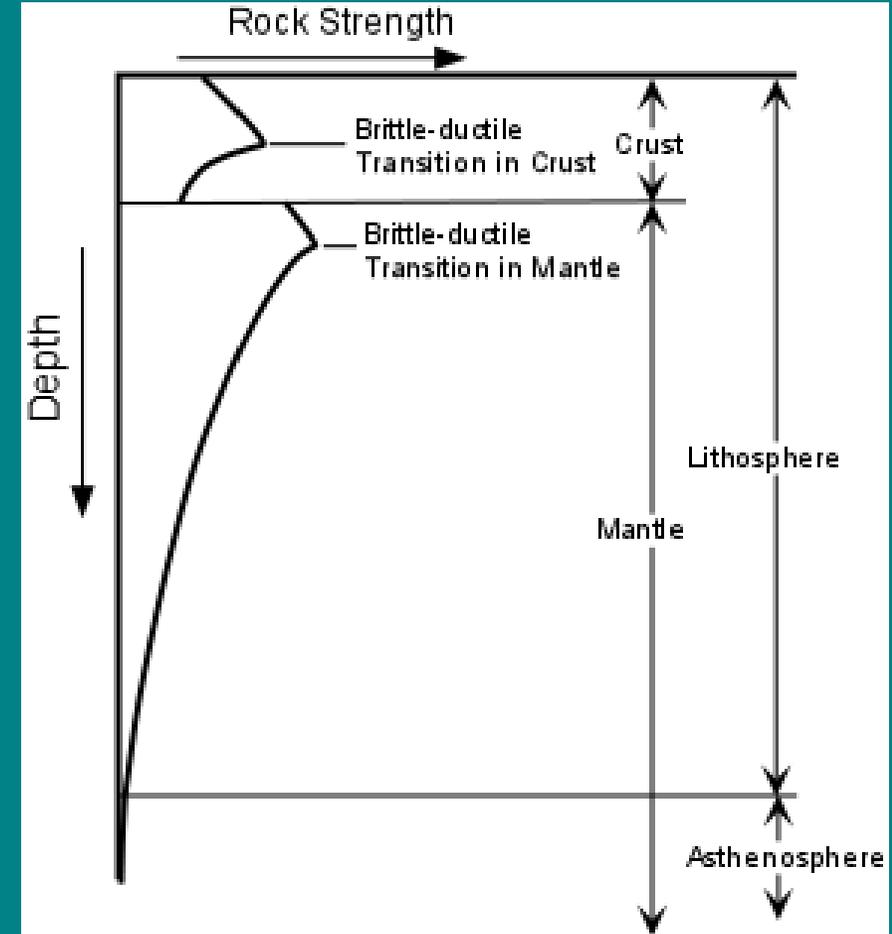
# Fractures



# Brittle vs. Ductile deformation

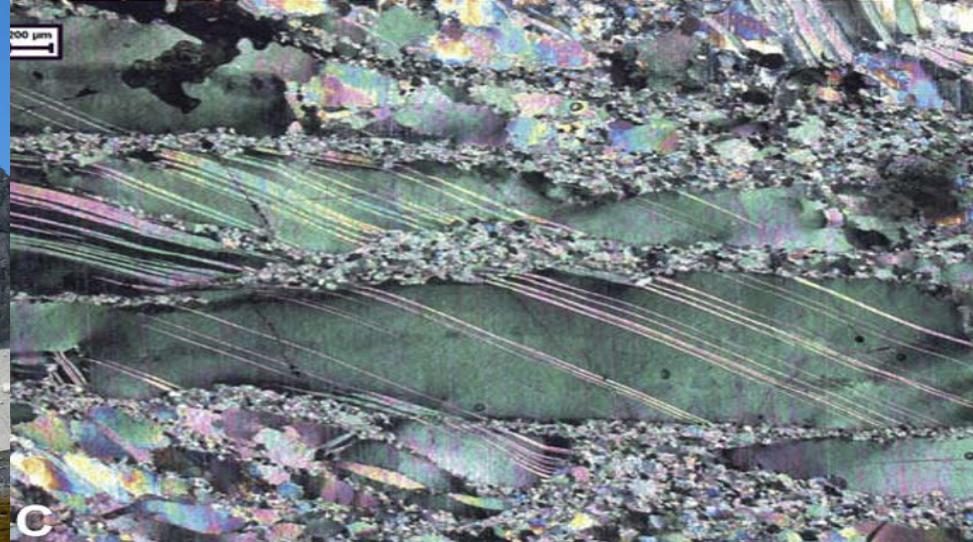
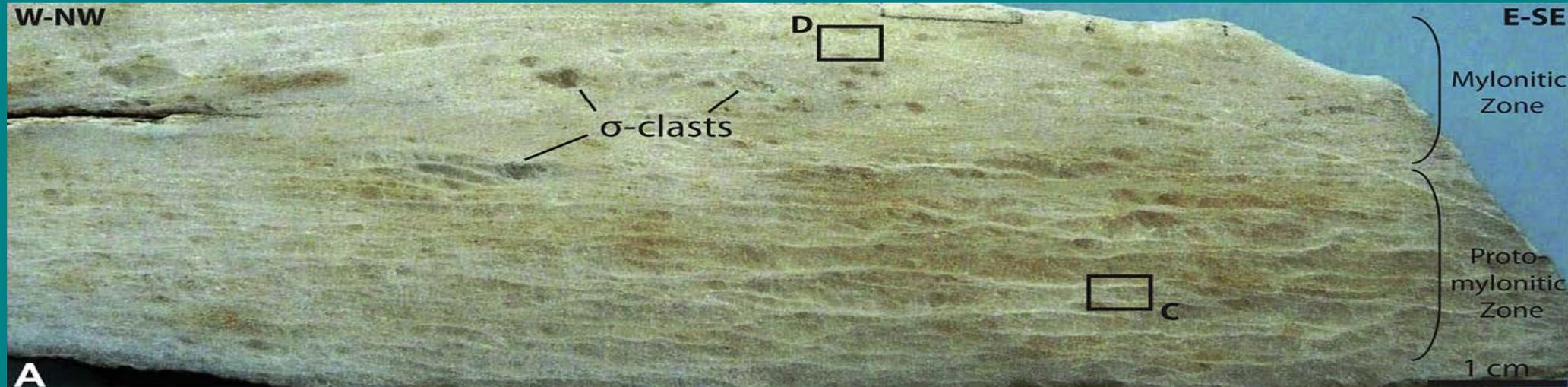
- Brittle

- Shallow in the Earth
- Relatively low temperature and pressure conditions
- Breakage of bonds
- Fracturing intense at low strain
- Strength is pressure dependent
- Time-independent
- Seismicity



<http://www.tulane.edu/~sanelson/eens1110/deform.htm>

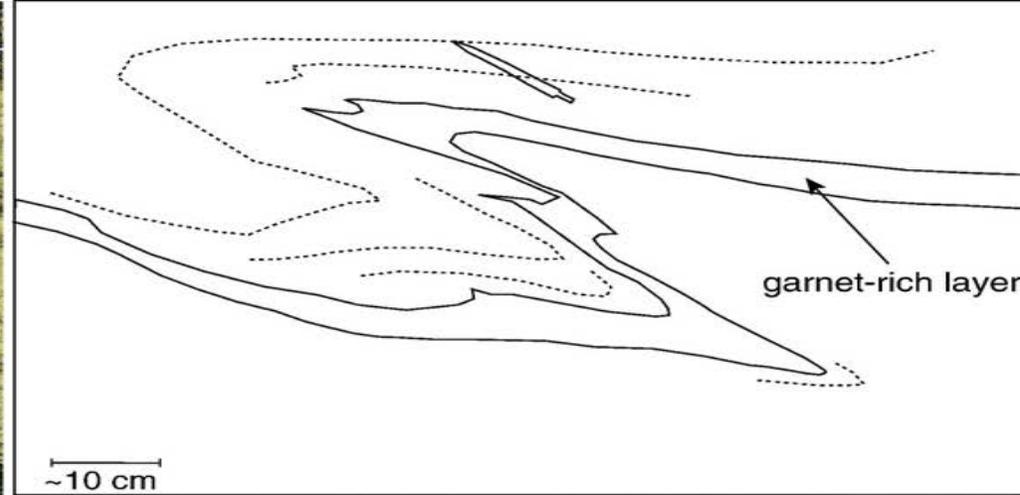
# Ductile deformation: Deformation structures in the field



# Ductile deformation: Deformation structures in the field

Mantle rocks in Norway: Barnhoorn et al., 2010

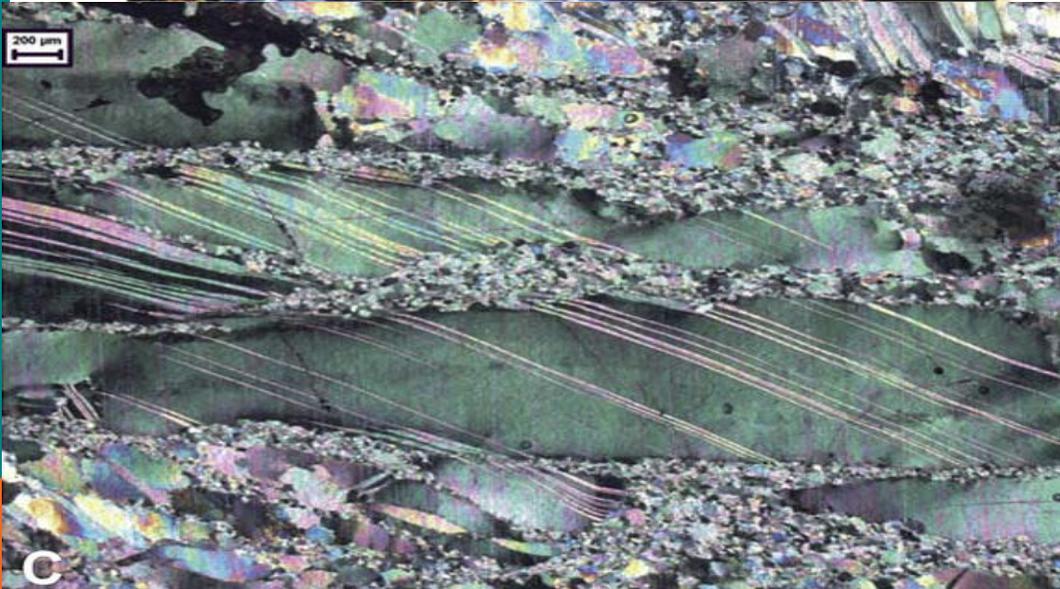
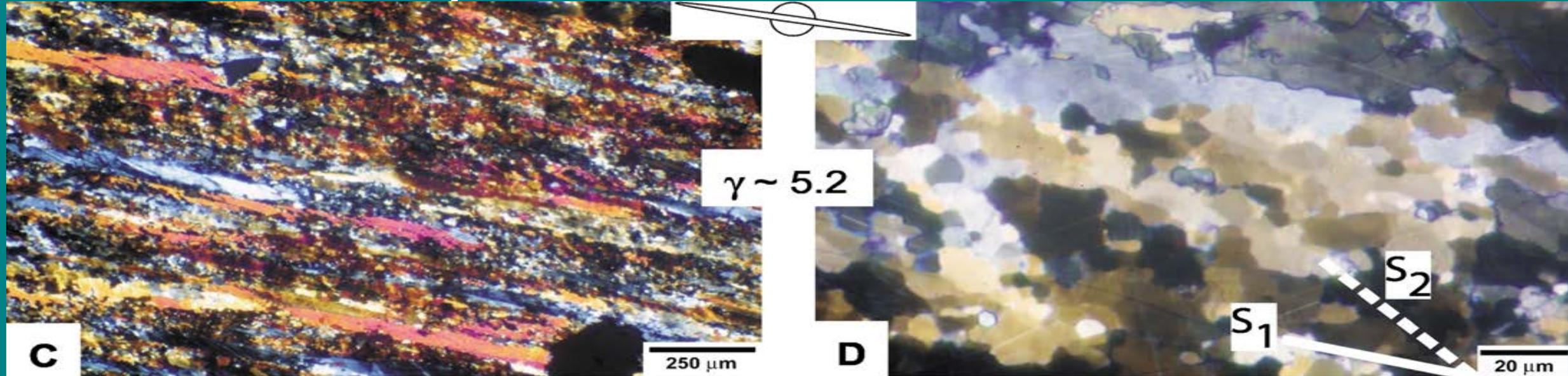
Relative viscosity  
contrast in folds and  
boudins



Crustal rocks



# Ductile deformation: Comparison field and experiment



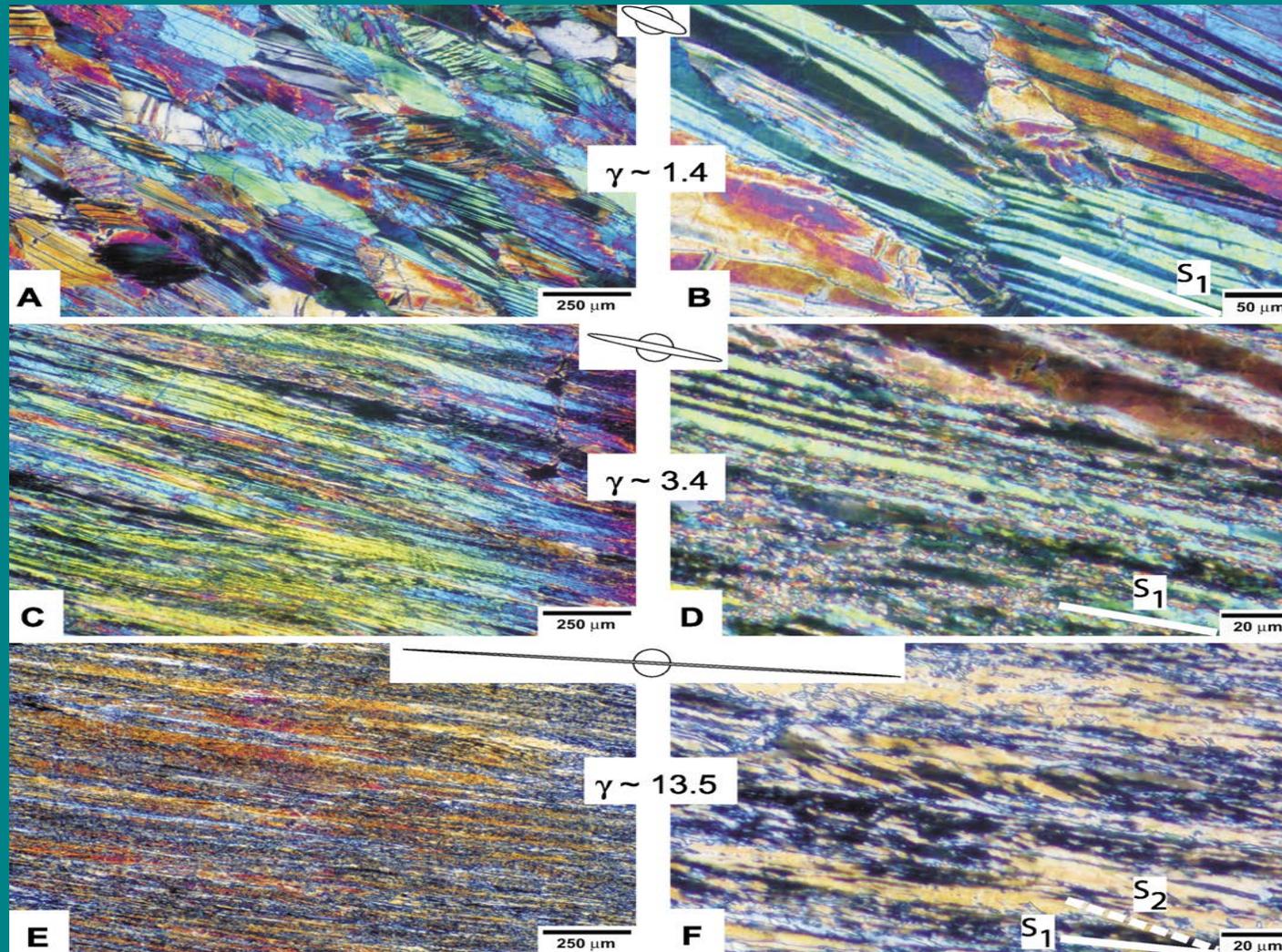
Lab and field: similar structures formed by similar deformation mechanisms

Barnhoorn et al., 2004

Lefebvre et al. 2011



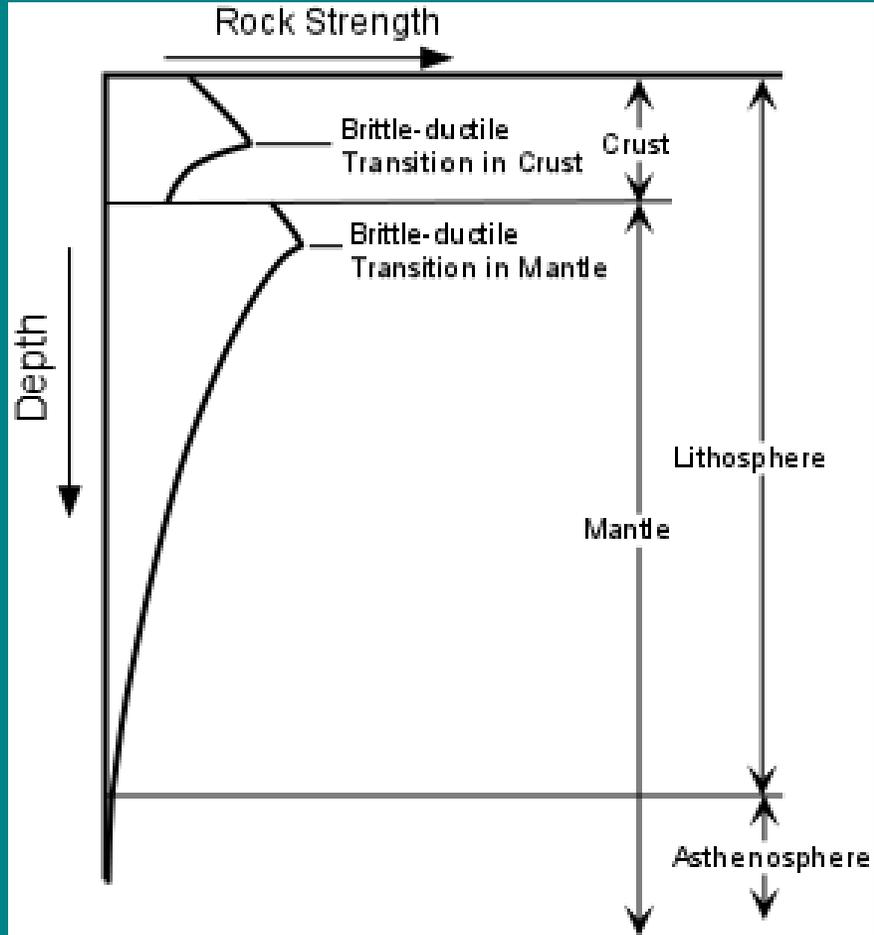
# Ductile shear zone experiments



Very high shear strains

Barnhoorn et al., 2004

# Brittle vs. Ductile deformation



<http://www.tulane.edu/~sanelson/eens1110/deform.htm>

- Ductile
  - Deep in the Earth
  - Relatively high temperature and pressure conditions
- Very high strain can be achieved without breakage of bonds
- Deformation primarily temperature controlled
- Time dependent process
- Occurs on geological time-scales

# Brittle vs. Ductile deformation

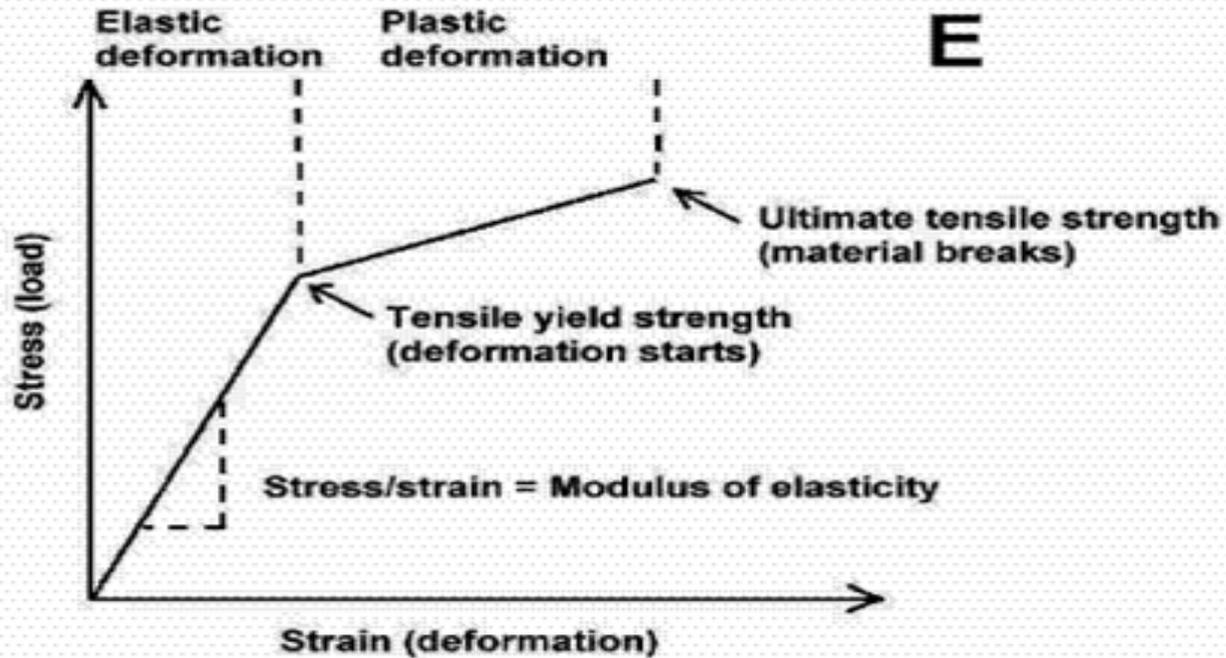
- Brittle

- Shallow in the Earth
- Relatively low temperature and pressure conditions
- Breakage of bonds
- Fracturing intense at low strain
- Strength is pressure dependent
- Time-independent
- Seismicity

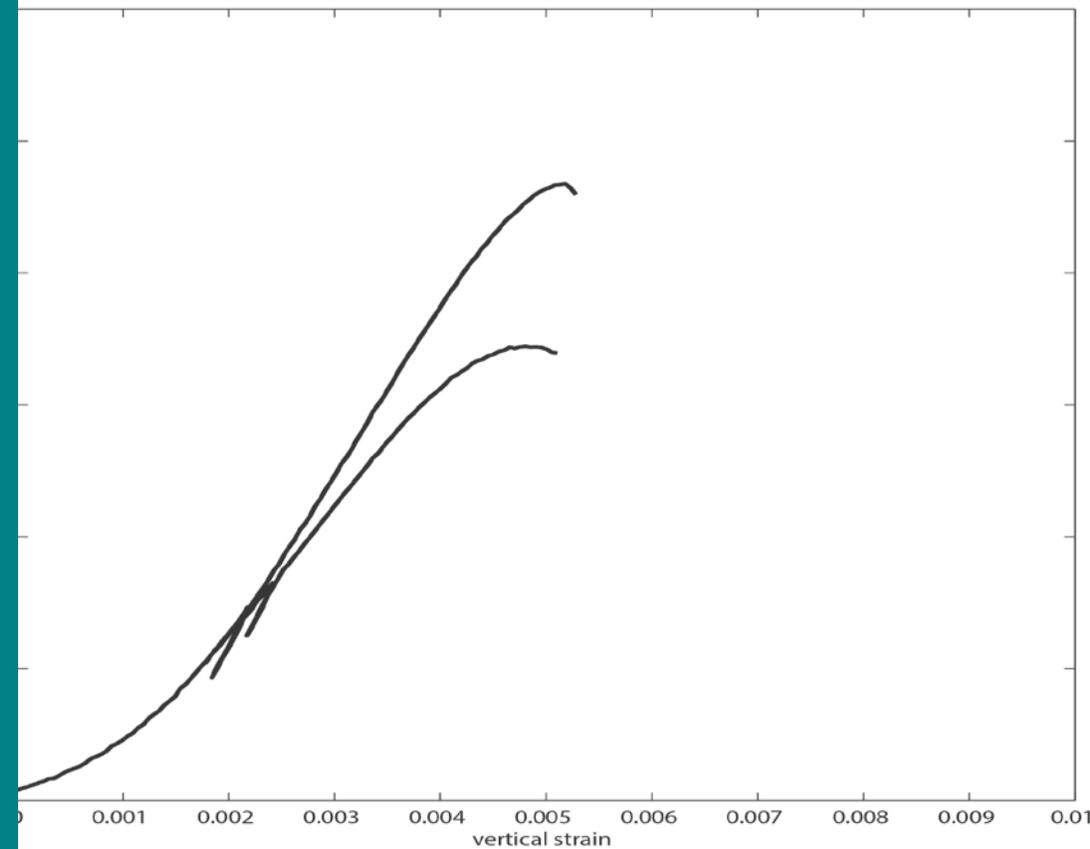
- Ductile

- Deep in the Earth
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- Very high strain can be achieved without breakage of bonds
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# Elasticity



sandstone



# Four Elastic Moduli

- Elastic moduli describe the elastic behaviour of a rock
  - Rock type dependent
  - Can vary with change in conditions (P,T)
- Used to predict & quantify elastic mechanical behaviour of rocks
  - Experiments
  - Geomechanical numerical modelling
  - Reservoir simulations
  - Etc.
- Wave propagation in seismics/geophysics can almost always be described by elasticity.

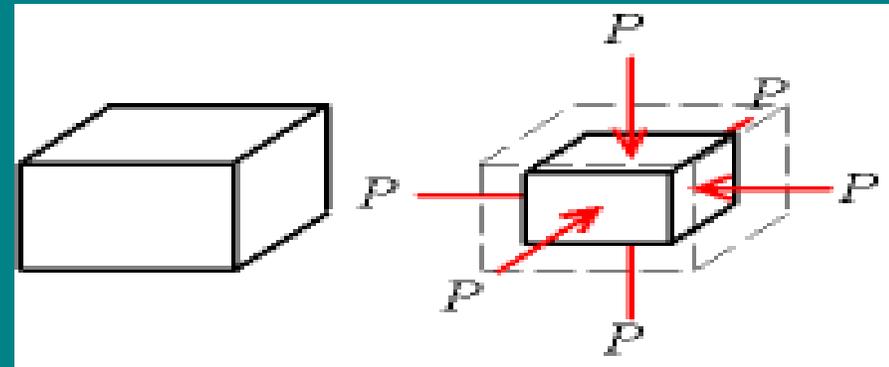


# Bulk modulus

The elastic mechanical behaviour of a material can completely be described by the bulk and shear modulus.

- The bulk modulus  $K_b$  [Pa] (also compression modulus) gives the resistance against deformation due to uniform compression and is defined by the hydrostatic pressure increase  $\Delta p$  that is necessary for a relative volume decrease  $\Delta V/V$ :

$$K_b = -V \frac{\partial p}{\partial V}$$



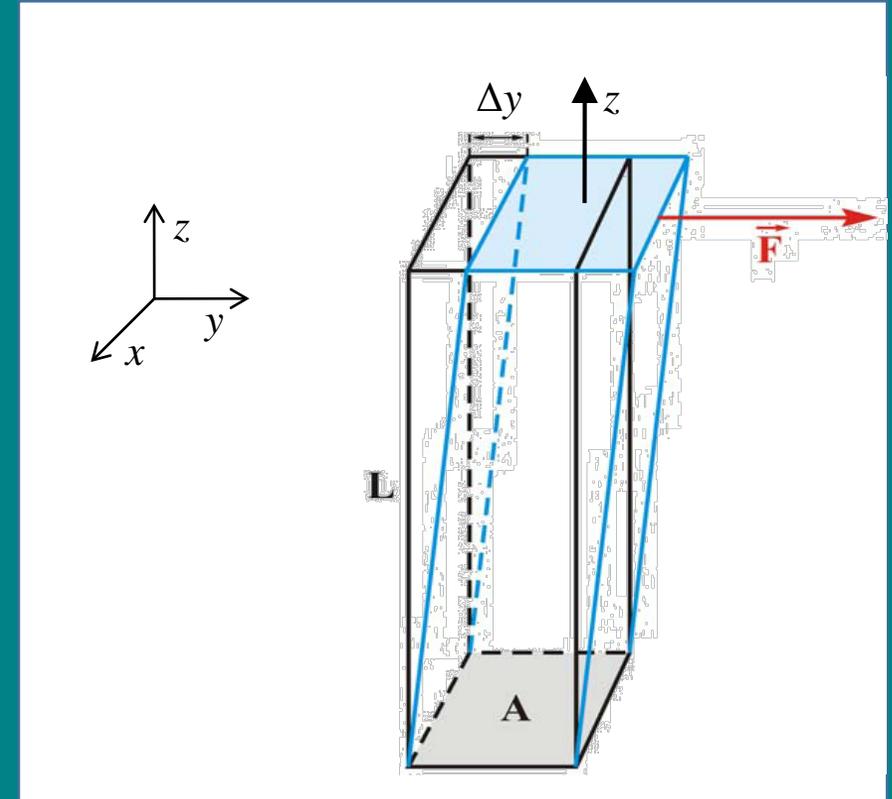
in which  $V$  is the initial volume that changes  $\Delta V$  due to  $\Delta p$ .

# Shear modulus

- The shear modulus  $\mu$  (or  $G$ ) [Pa] gives the resistance against deformation due to shearing and is defined by the shear strain  $\varepsilon_{ij}$  as a result of a shearing stress  $\sigma_{ij}$ .  
For example:

in which 
$$\mu = \frac{\sigma_{zy}}{2\varepsilon_{zy}}$$

- the shear stress  $\sigma_{zy} = F/A$  is the force  $F$  in the  $y$ -direction per area  $A$  with normal  $z$ .
- the shear strain  $\varepsilon_{zy} = \Delta y/L$  is the transversal displacement  $\Delta y$  over the initial length  $L$  perpendicular to  $A$  (in the  $z$ -direction).



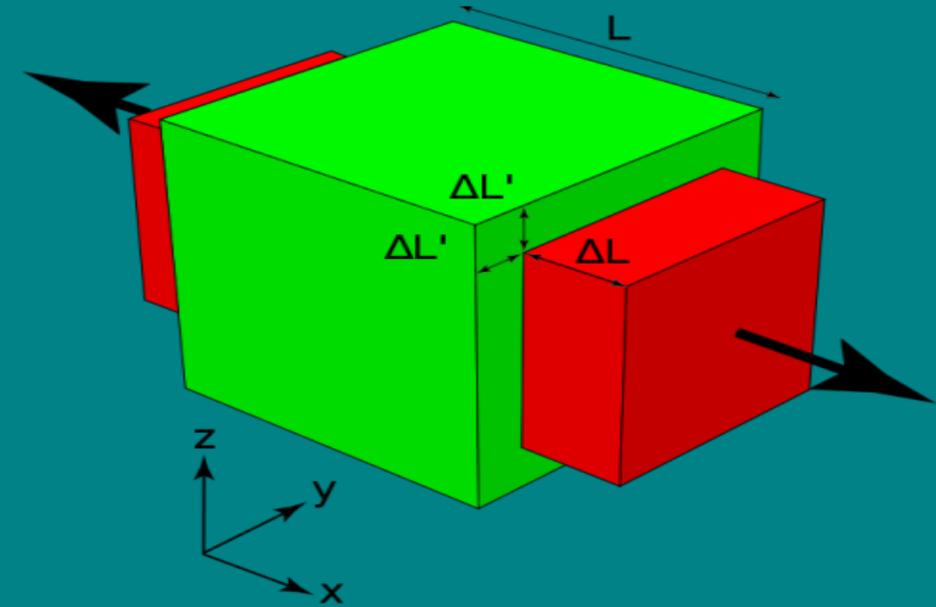
# Young's modulus

Young's modulus  $E$  [Pa] is a measure of the stiffness of the material and is given by the ratio of uniaxial stress  $\sigma_{xx}$  [Pa] and uniaxial strain  $\varepsilon_{xx}$  [-].

with 
$$E = \frac{\sigma_{xx}}{\varepsilon_{xx}}$$

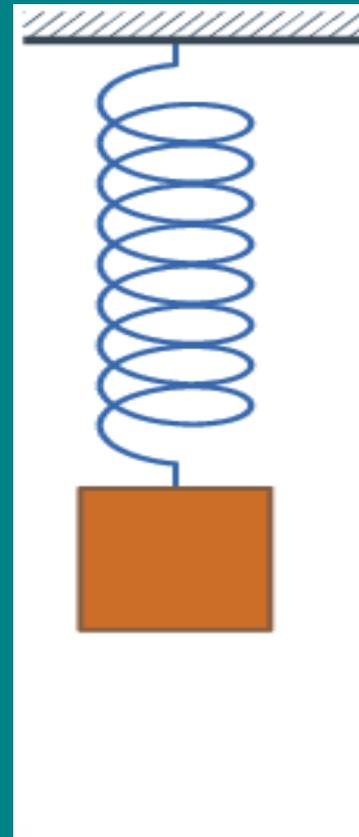
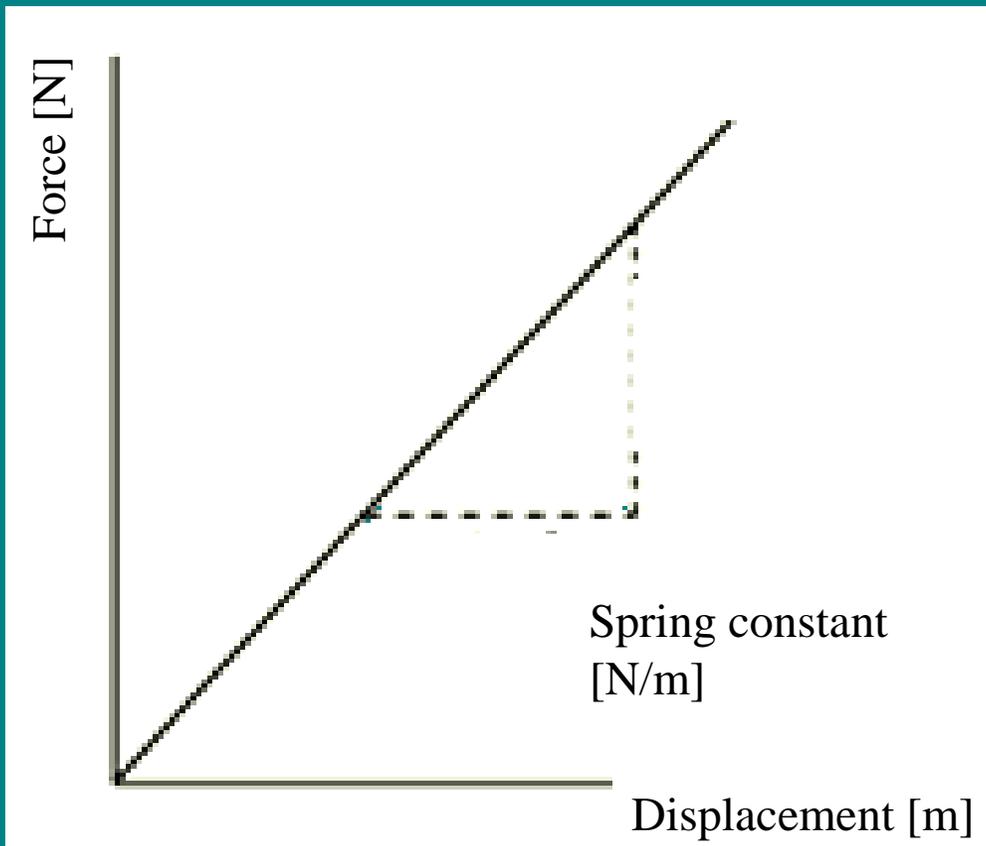
-  $\sigma_{xx} = F/A$  is the force  $F$  in the  $x$ -direction per area  $A$  with the normal in the  $x$ -direction.

-  $\varepsilon_{xx} = \Delta L/L$  is the relative change in length (tension or compression) in the  $x$ -direction due to a force in the  $x$ -direction.



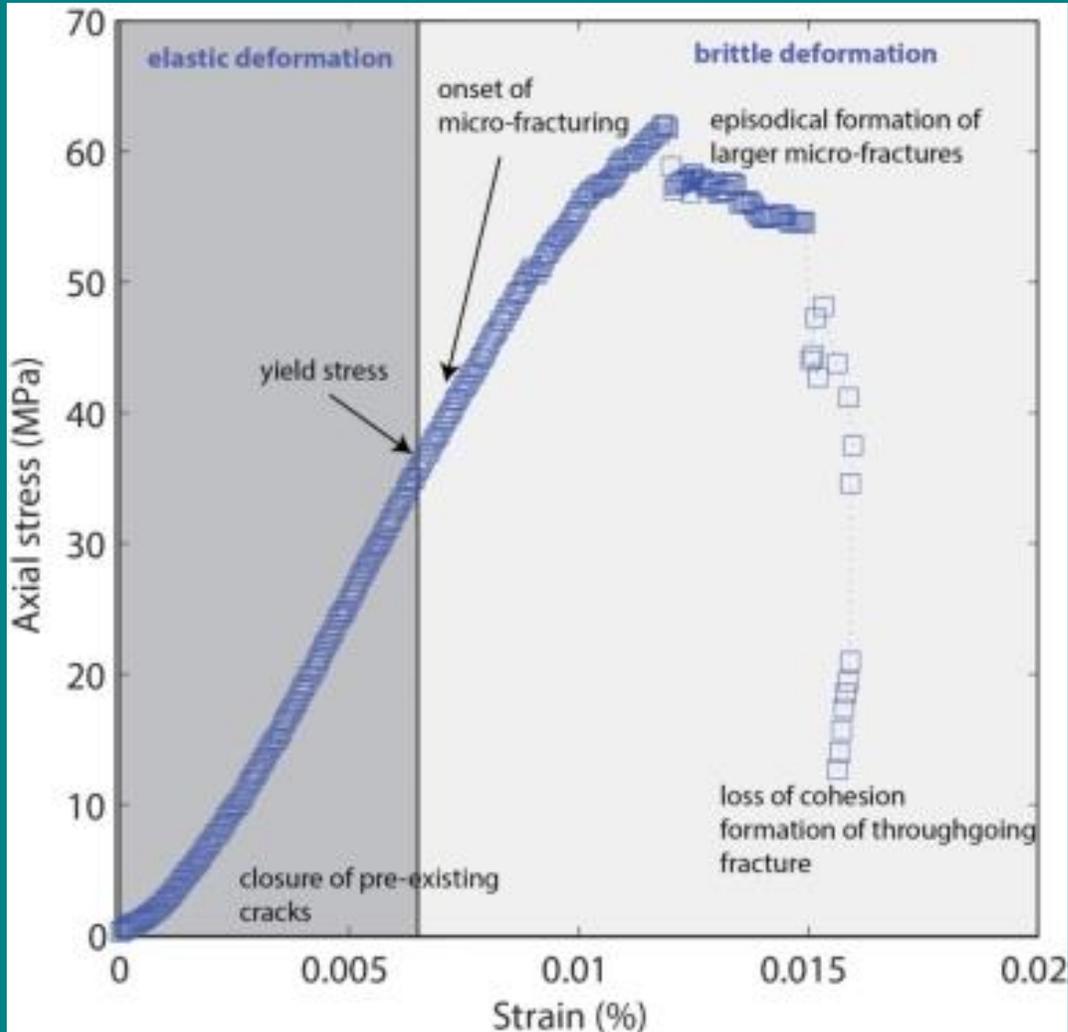
# Hooke's law for an isotropic elastic solid

Hooke's law states that the strain  $\varepsilon$  ( $= \Delta L/L$ ) in an isotropic elastic solid in 1D is proportional to the stress (force per unit area) via the Young's modulus  $E$ .



$$\sigma = E\varepsilon$$

# In-class stress-strain exercise - 3



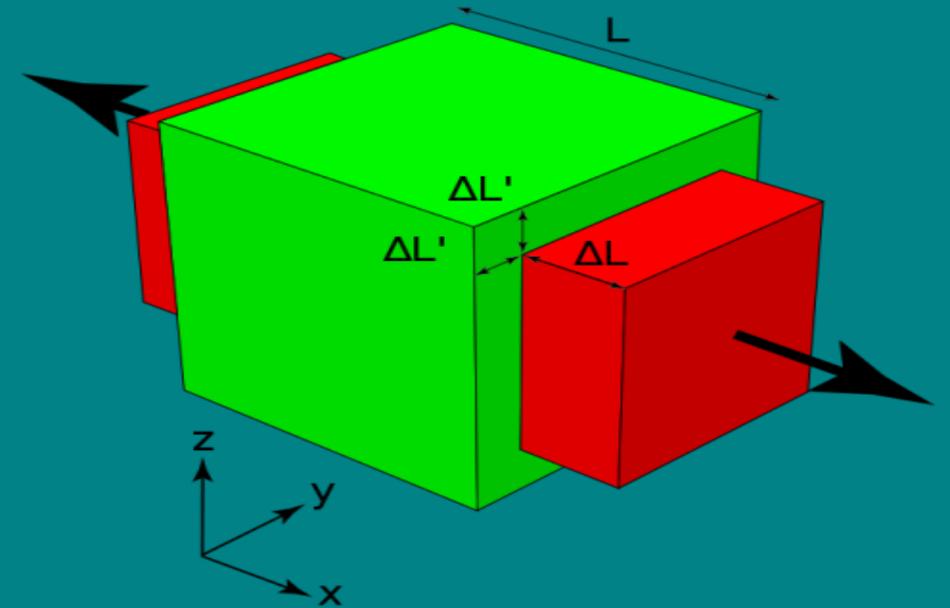
- What is the stress-strain ratio ( $\Delta\sigma/\Delta\varepsilon$ ) of this experimentally deformed shale sample? Determine the stress-strain ratio only in the gray part of the diagram. The unit of the stress-strain ratio is in GPa.

# Poisson's ratio

Poisson's ratio  $\nu$  [-] describes the strain in the transversal direction (e.g.  $y$  and  $z$ ), which originates from tension (stretching) or compression in the axial direction (e.g.  $x$ ).

with 
$$\nu = - \frac{\varepsilon_{yy}}{\varepsilon_{xx}}$$

- $\varepsilon_{xx} = \Delta L/L$  is the uniaxial strain.
- $\varepsilon_{yy} = \Delta L'/L'$  is the transversal (or lateral) strain.



# Elastic relationships in isotropic materials

$K$	$E$	$\lambda$	$\nu$	$M$	$\mu$
$\lambda + 2\mu/3$	$\mu \frac{3\lambda + 2\mu}{\lambda + \mu}$	—	$\frac{\lambda}{2(\lambda + \mu)}$	$\lambda + 2\mu$	—
—	$9K \frac{K - \lambda}{3K - \lambda}$	—	$\frac{\lambda}{3K - \lambda}$	$3K - 2\lambda$	$3(K - \lambda)/2$
—	$\frac{9K\mu}{3K + \mu}$	$K - 2\mu/3$	$\frac{3K - 2\mu}{2(3K + \mu)}$	$K + 4\mu/3$	—
$\frac{E\mu}{3(3\mu - E)}$	—	$\mu \frac{E - 2\mu}{(3\mu - E)}$	$E/(2\mu) - 1$	$\mu \frac{4\mu - E}{3\mu - E}$	—
—	—	$3K \frac{3K - E}{9K - E}$	$\frac{3K - E}{6K}$	$3K \frac{3K + E}{9K - E}$	$\frac{3KE}{9K - E}$
$\lambda \frac{1 + \nu}{3\nu}$	$\lambda \frac{(1 + \nu)(1 - 2\nu)}{\nu}$	—	—	$\lambda \frac{1 - \nu}{\nu}$	$\lambda \frac{1 - 2\nu}{2\nu}$
$\mu \frac{2(1 + \nu)}{3(1 - 2\nu)}$	$2\mu(1 + \nu)$	$\mu \frac{2\nu}{1 - 2\nu}$	—	$\mu \frac{2 - 2\nu}{1 - 2\nu}$	—
—	$3K(1 - 2\nu)$	$3K \frac{\nu}{1 + \nu}$	—	$3K \frac{1 - \nu}{1 + \nu}$	$3K \frac{1 - 2\nu}{2 + 2\nu}$
$\frac{E}{3(1 - 2\nu)}$	—	$\frac{E\nu}{(1 + \nu)(1 - 2\nu)}$	—	$\frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}$	$\frac{E}{2 + 2\nu}$

Mavko, 2003

Determine Poisson's ratio using  $K_b$  and  $\mu$

$$\nu = \frac{3K_b - 2\mu}{2(3K_b + \mu)}$$

Determine Young's modulus using  $K_b$  and  $\mu$

$$E = \frac{9K_b\mu}{3K_b + \mu}$$

*Message: if you have 2 elastic constants, you can calculate all 4!!*



# Elastic relationships in isotropic materials

$K$	$E$	$\lambda$	$\nu$	$M$	$\mu$
$\lambda + 2\mu/3$	$\mu \frac{3\lambda + 2\mu}{\lambda + \mu}$	—	$\frac{\lambda}{2(\lambda + \mu)}$	$\lambda + 2\mu$	—
—	$9K \frac{K - \lambda}{3K - \lambda}$	—	$\frac{\lambda}{3K - \lambda}$	$3K - 2\lambda$	$3(K - \lambda)/2$
—	$\frac{9K\mu}{3K + \mu}$	$K - 2\mu/3$	$\frac{3K - 2\mu}{2(3K + \mu)}$	$K + 4\mu/3$	—
$\frac{E\mu}{3(3\mu - E)}$	—	$\mu \frac{E - 2\mu}{(3\mu - E)}$	$E/(2\mu) - 1$	$\mu \frac{4\mu - E}{3\mu - E}$	—
—	—	$3K \frac{3K - E}{9K - E}$	$\frac{3K - E}{6K}$	$3K \frac{3K + E}{9K - E}$	$\frac{3KE}{9K - E}$
$\lambda \frac{1 + \nu}{3\nu}$	$\lambda \frac{(1 + \nu)(1 - 2\nu)}{\nu}$	—	—	$\lambda \frac{1 - \nu}{\nu}$	$\lambda \frac{1 - 2\nu}{2\nu}$
$\mu \frac{2(1 + \nu)}{3(1 - 2\nu)}$	$2\mu(1 + \nu)$	$\mu \frac{2\nu}{1 - 2\nu}$	—	$\mu \frac{2 - 2\nu}{1 - 2\nu}$	—
—	$3K(1 - 2\nu)$	$3K \frac{\nu}{1 + \nu}$	—	$3K \frac{1 - \nu}{1 + \nu}$	$3K \frac{1 - 2\nu}{2 + 2\nu}$
$\frac{E}{3(1 - 2\nu)}$	—	$\frac{E\nu}{(1 + \nu)(1 - 2\nu)}$	—	$\frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}$	$\frac{E}{2 + 2\nu}$

Mavko, 2003

# Examples of earth's materials

Mineral	Density	Young's Modulus	Bulk Modulus	Shear Modulus	Vp	Vs	Poisson's Ratio
Quartz	2.6500	95.756	36.600	45.000	6.0376	4.1208	0.063953
Calcite	2.7100	84.293	76.800	32.000	6.6395	3.4363	0.31707
Dolomite	2.8700	116.57	94.900	45.000	7.3465	3.9597	0.29527
Clay (kaolinite)	1.5800	3.2034	1.5000	1.4000	1.4597	0.94132	0.14407
Muscovite	2.7900	100.84	61.500	41.100	6.4563	3.8381	0.22673
Feldspar (Albite)	2.6300	69.010	75.600	25.600	6.4594	3.1199	0.34786
Halite	2.1600	37.242	24.800	14.900	4.5474	2.6264	0.24972
Anhydrite	2.9800	74.431	56.100	29.100	5.6432	3.1249	0.27888
Pyrite	4.9300	305.85	147.40	132.50	8.1076	5.1842	0.15417
Siderite	3.9600	134.51	123.70	51.000	6.9576	3.5887	0.31876
gas	0.00065000	0.0000	0.00013000	0.0000	0.44721	0.0000	0.50000
water	1.0000	0.0000	2.2500	0.0000	1.5000	0.0000	0.50000
oil	0.80000	0.0000	1.0200	0.0000	1.1292	0.0000	0.50000

Mavko, 2003



# Modulus of Elasticity of Some Common Rocks

	Name of Rocks	Youngs Modulus (E), Gpa
Igneous Rocks	Basalt	20 – 100
	Diabase	30 – 90
	Gabbro	60 – 110
	Granite	26 – 70
	Syemite	60 – 80
	Dolomite	20 – 44
Sedimentary Rocks	Limestone	10 – 80
	Sandstone	5 – 86
	Shale	08 – 30
	Gneiss	20 – 60
Metamorphic Rocks	Marble	60 – 90
	Quartzite	26 – 102
	Schist	41 – 72

<http://civilblog.org/2015/02/13/what-are-the-values-of-modulus-of-elasticity-poissons-ratio-for-different-rocks/>

# Values of Poisson's Ratio for Some Common Rocks

Types of Rocks	Name of Rocks	Average Values of Poisson's Ratio ( $\nu$ )
Igneous Rocks	Basalt	0.14 – 0.20
	Diabase	0.125 – 0.25
	Gabbro	0.125 – 0.25
	Granite	0.125 – 0.25
	Syemite	0.25
Sedimentary Rocks	Dolomite	0.08 – 0.20
	Limestone	0.10 – 0.20
	Sandstone	0.066 – 0.125
	Shale	0.11 – 0.54
Metamorphic Rocks	Gneiss	0.091 – 0.25
	Marble	0.25 – 0.38
	Quartzite	0.23
	Schist	0.01 – 0.20

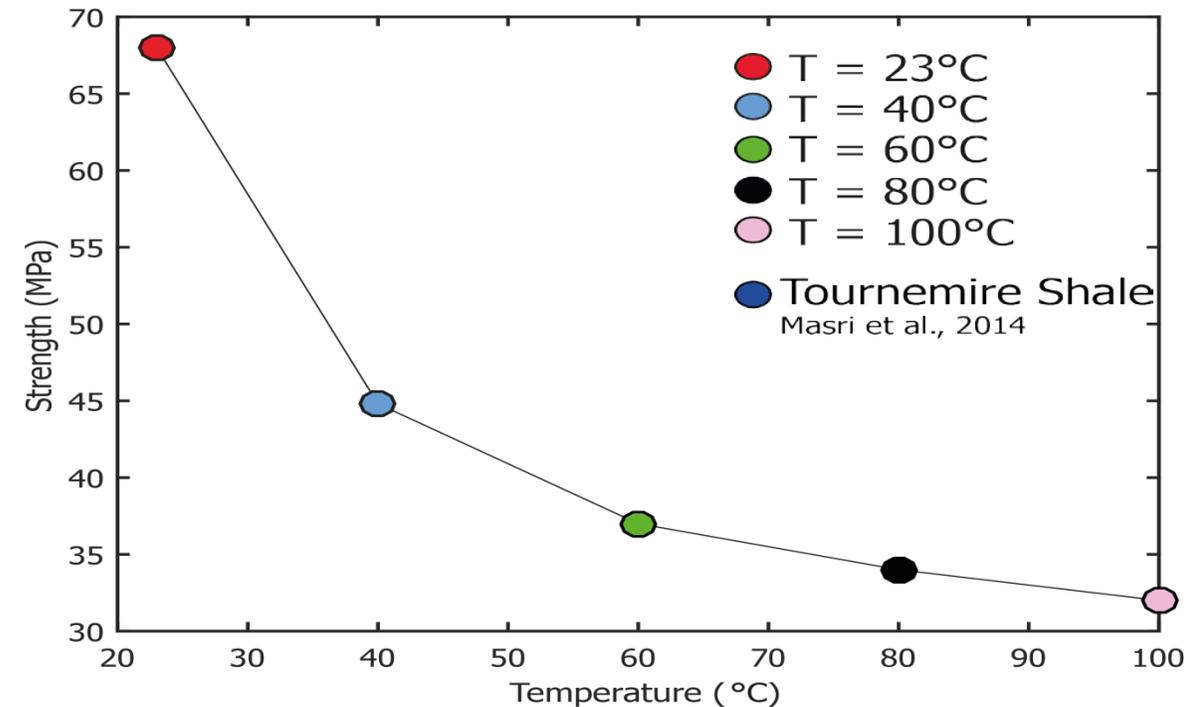
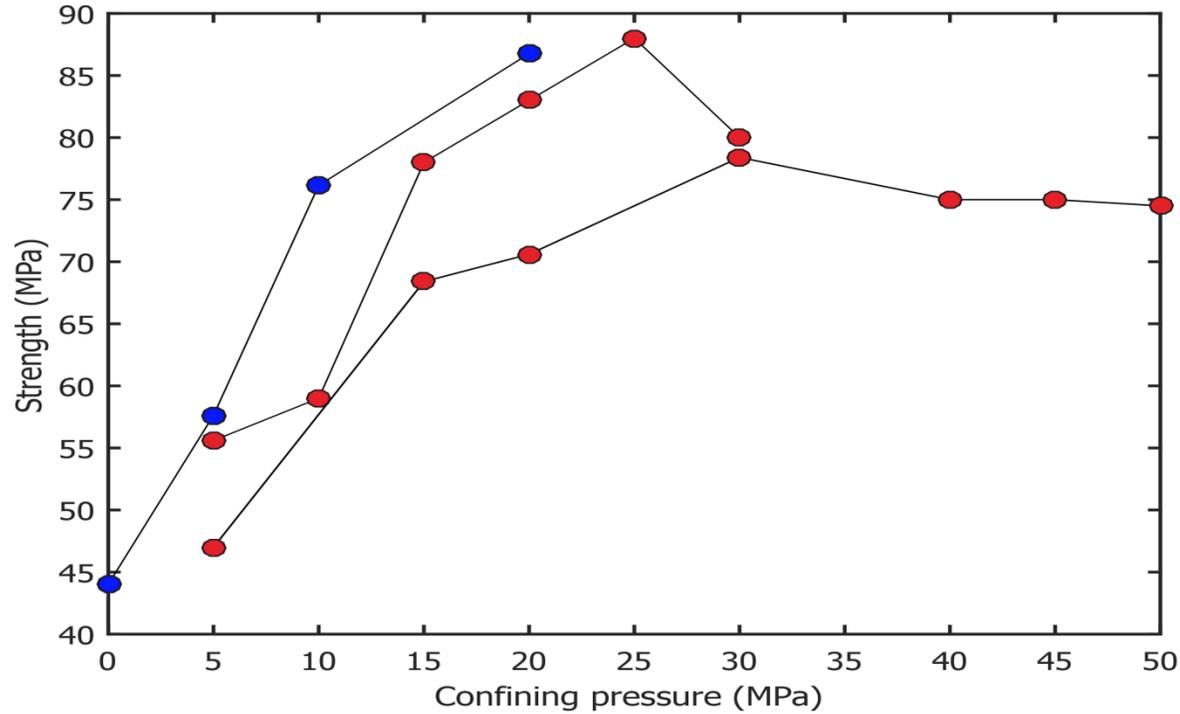
<http://civilblog.org/2015/02/13/what-are-the-values-of-modulus-of-elasticity-poissons-ratio-for-different-rocks/>

# Elastic moduli

- If you know 2 elastic moduli (for isotropic materials) you can calculate all other moduli
- And thus fully describe the elastic behaviour of the material
  - Geomechanical reservoir simulations
  - Hydraulic fracturing
  - Seismicity studies

# Important!!!

## Rock mechanical constant change with depth and temperature



unpublished TU Delft work on shales

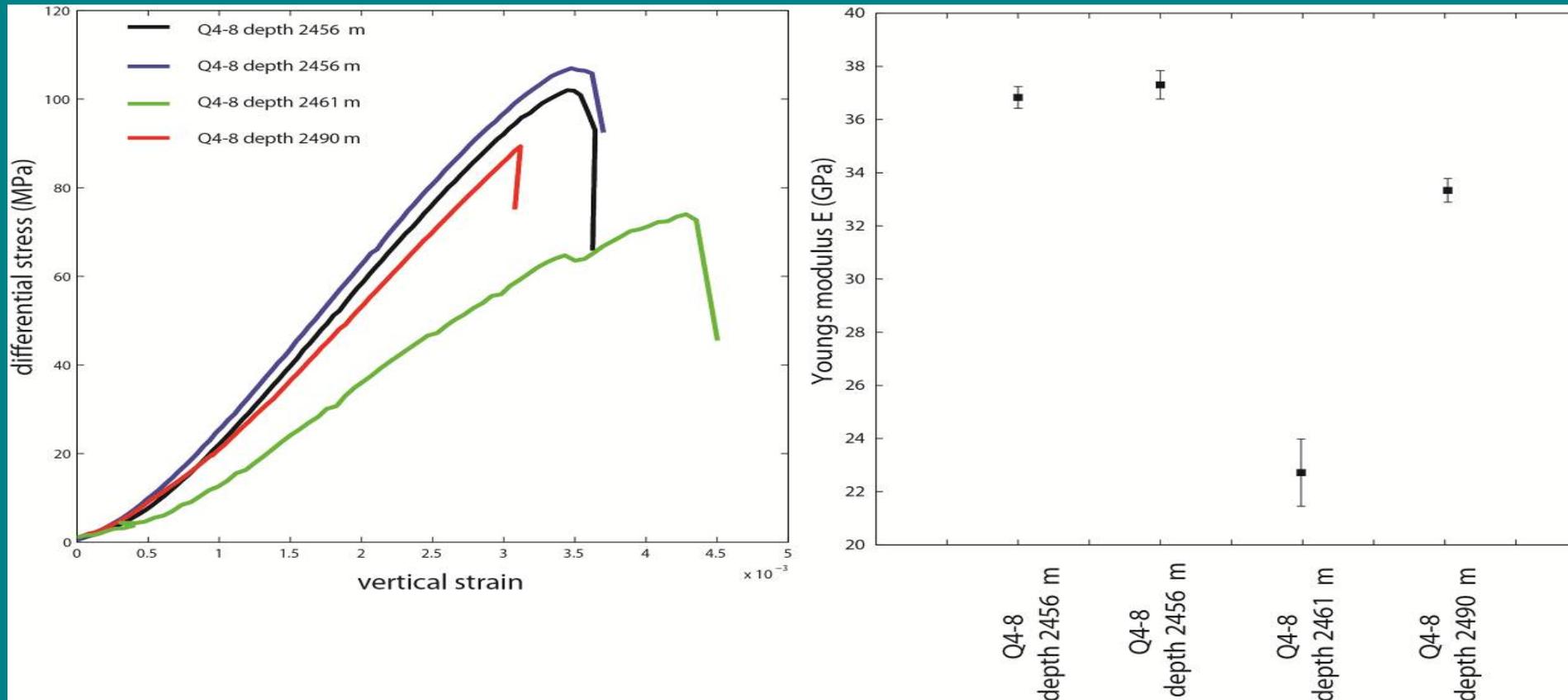
$P_{conf} = 15 \text{ MPa}$

22/01/2019

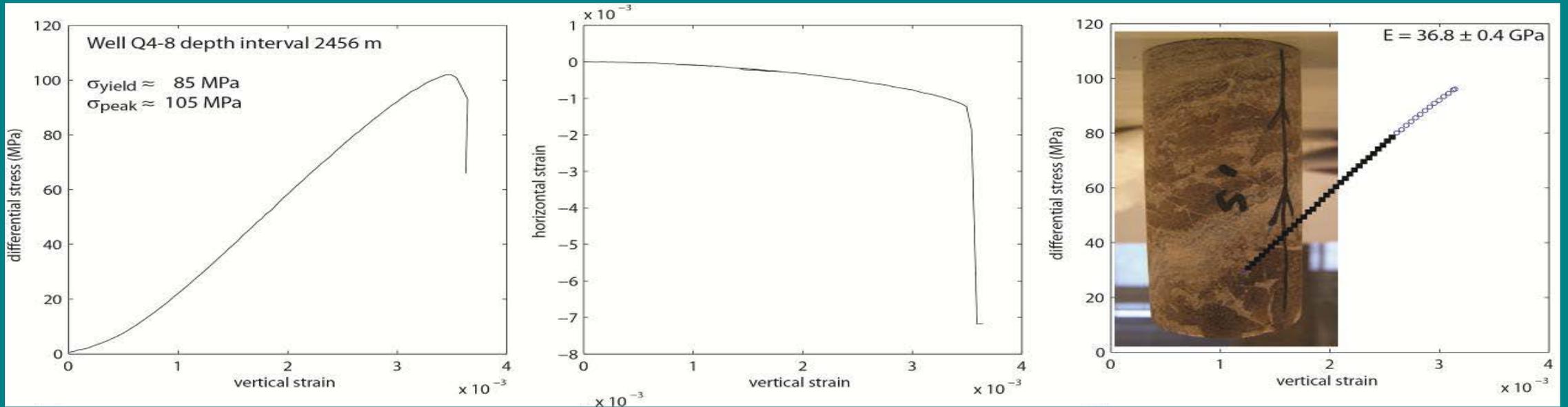


75

# Rock mechanics: Shaley sandstone



Measurement of Stress, Horizontal strain and Vertical strain



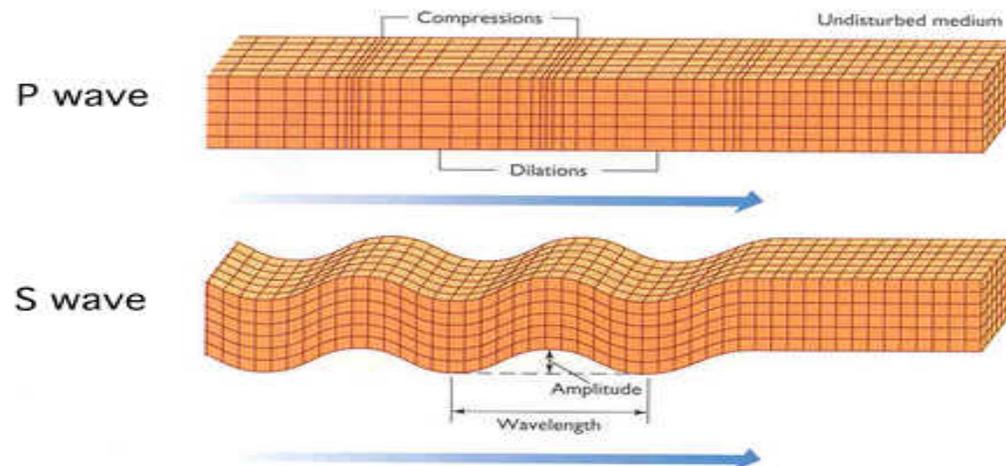
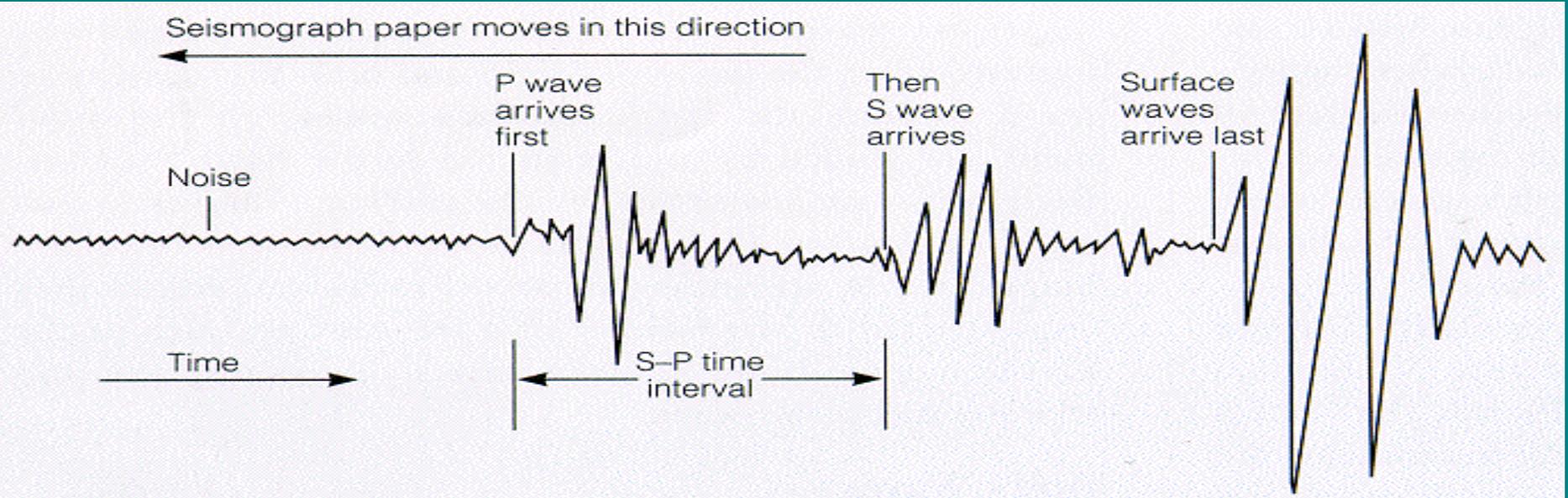
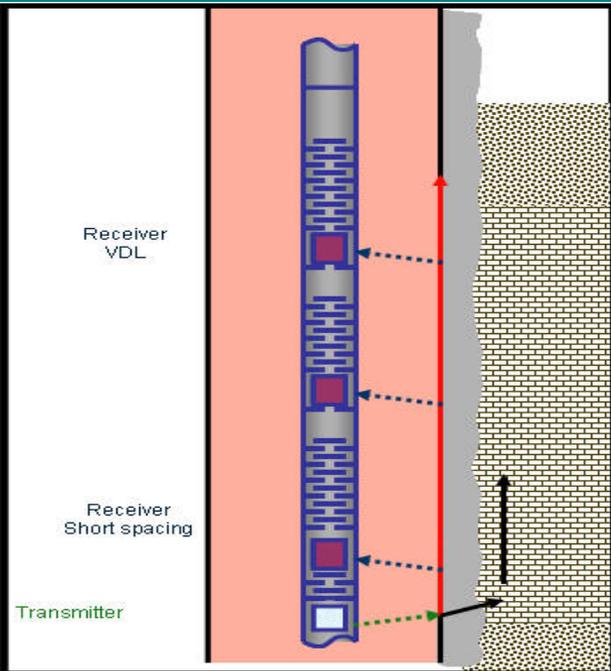
$$E = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{9K_b\mu}{3K_b + \mu}$$

$$\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \frac{3K_b - 2\mu}{2(3K_b + \mu)}$$

# Acoustics logging



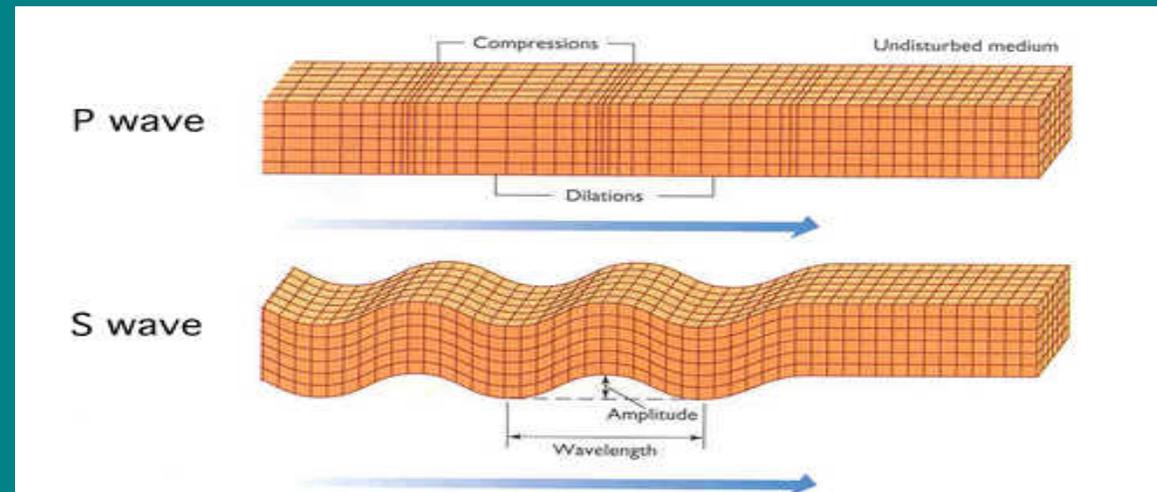
# Seismic waves



# Vp and Vs

$$\mu = v_s^2 \rho$$

$$K_b = \left( v_p^2 - \frac{4}{3} v_s^2 \right) \rho$$



Measurement of Vp, Vs and density

# Vp and Vs

- You can calculate the P- and S-wave velocities for a specific rock type if the elastic moduli are known

$$V_p = \sqrt{\frac{K_b + \frac{4}{3}\mu}{\rho}} = \sqrt{\frac{\lambda + 2\mu}{\rho}} =$$

$$V_s = \sqrt{\frac{\mu}{\rho}} =$$

# Poisson's ratio

$$\nu = \frac{v_p^2 - 2v_s^2}{2(v_p^2 - v_s^2)}$$

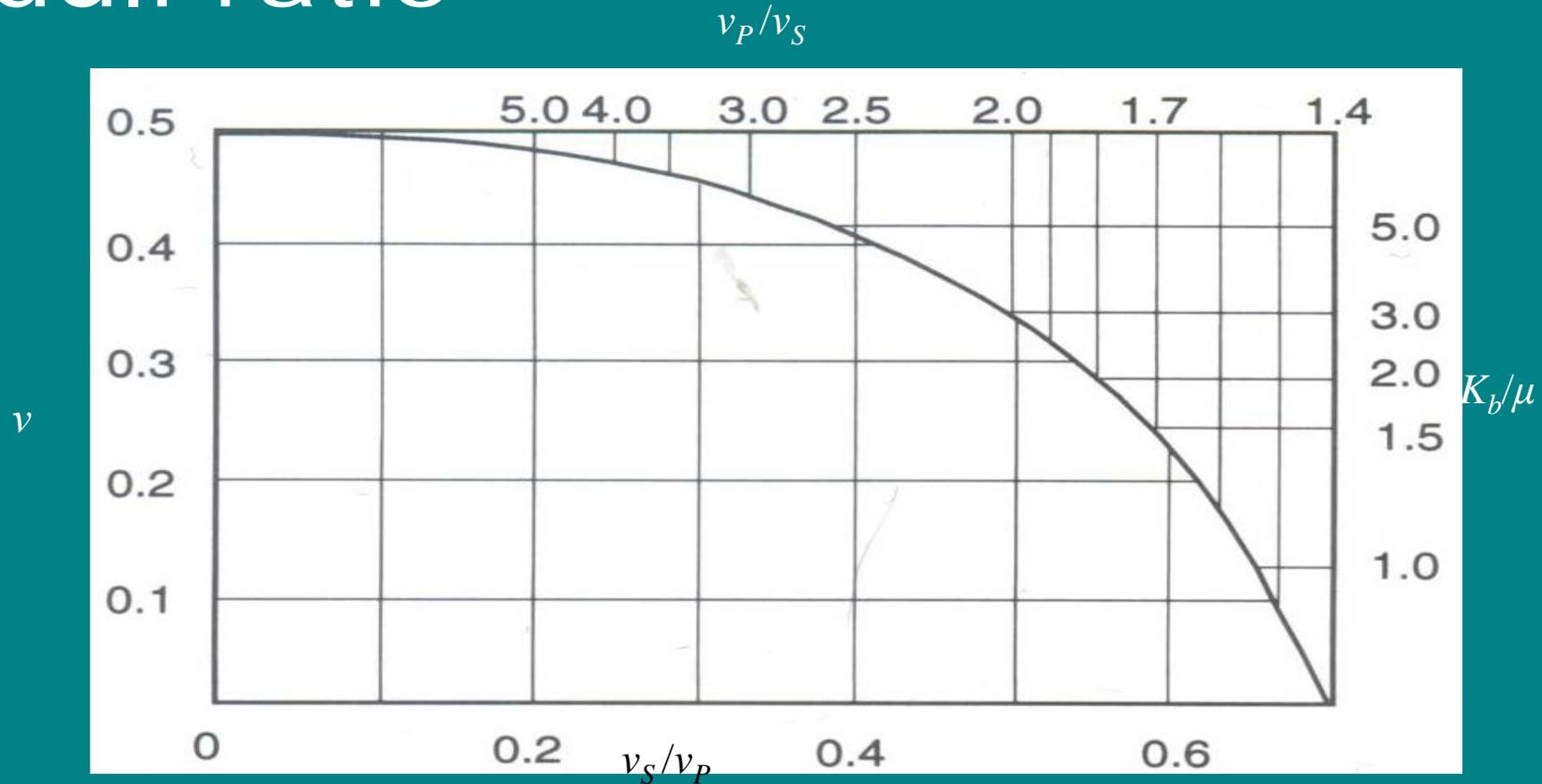
Because the elastic constants are always positive,  $v_p$  is always greater than  $v_s$ , and

$$\frac{v_s}{v_p} = \sqrt{\frac{\mu}{\lambda + 2\mu}} = \sqrt{\frac{0.5 - \nu}{1 - \nu}}$$

As  $\nu$  decreases from  $1/2$  to  $0$ ,  $v_s/v_p$  increases from  $0$  to its maximum value  $1/\sqrt{2}$ .

Thus the velocity of the S-wave ranges from  $0$  to  $70\%$  of the velocity of the P-wave.

# Velocity ratio, Poisson's ratio, and moduli ratio



Sheriff and Geldart, 1995

# In-class elastic moduli exercise - 10 min

- Sandstone  $\rho = 2.65 \text{ g/cm}^3$ ,  $V_p = 5491 \text{ m/s}$ .  $V_s = 3463 \text{ m/s}$
- Limestone  $\rho = 2.71 \text{ g/cm}^3$ ,  $V_p = 6417 \text{ m/s}$ .  $V_s = 3444 \text{ m/s}$
- 
  
- Wat are  $K_b$ ,  $\mu$ ,  $E$  and  $\nu$  for these 2 rock types?



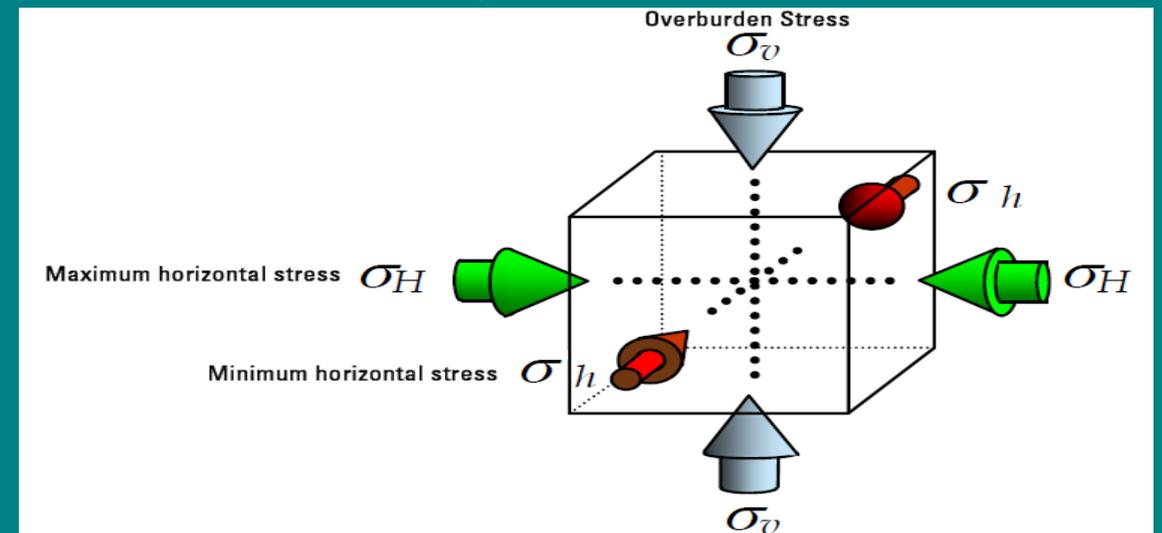
# 15 minute break

# Stresses in the subsurface

**Vertical or Overburden Stress ( $\sigma_v$ )** Which often has the greatest magnitude. It is also the maximum in-situ stress.

**Maximum Horizontal Stress ( $\sigma_H$ )** which is the intermediate in situ stress.

**Minimum Horizontal Stress ( $\sigma_h$ )** which has the lowest magnitude. It is also the minimum in-situ stress.



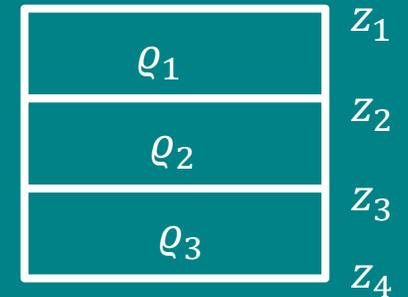
# Stresses from borehole logs

- Vertical stress ( $\sigma_v$ )

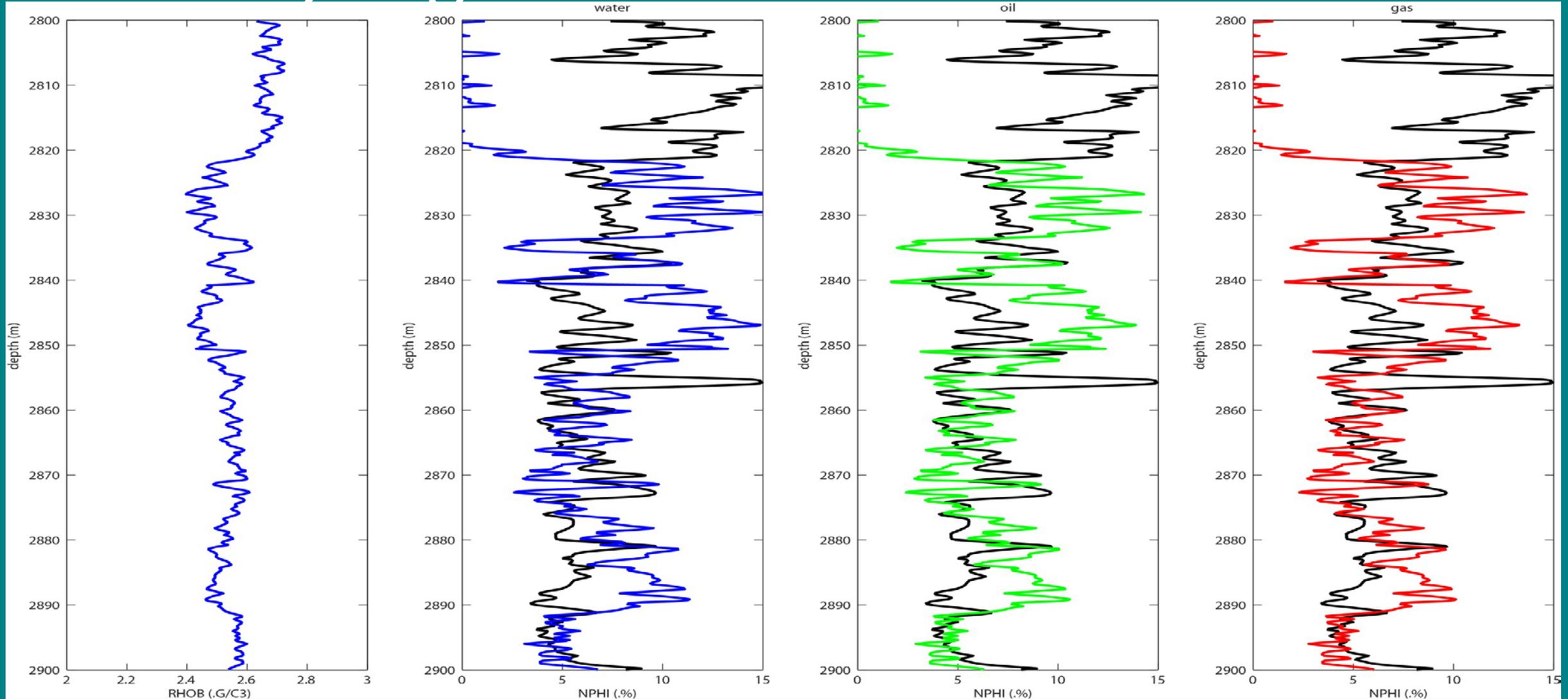
$$\sigma_v = \rho g z$$

- Density information or density logs
  - Summation of all density + heights of each lithology

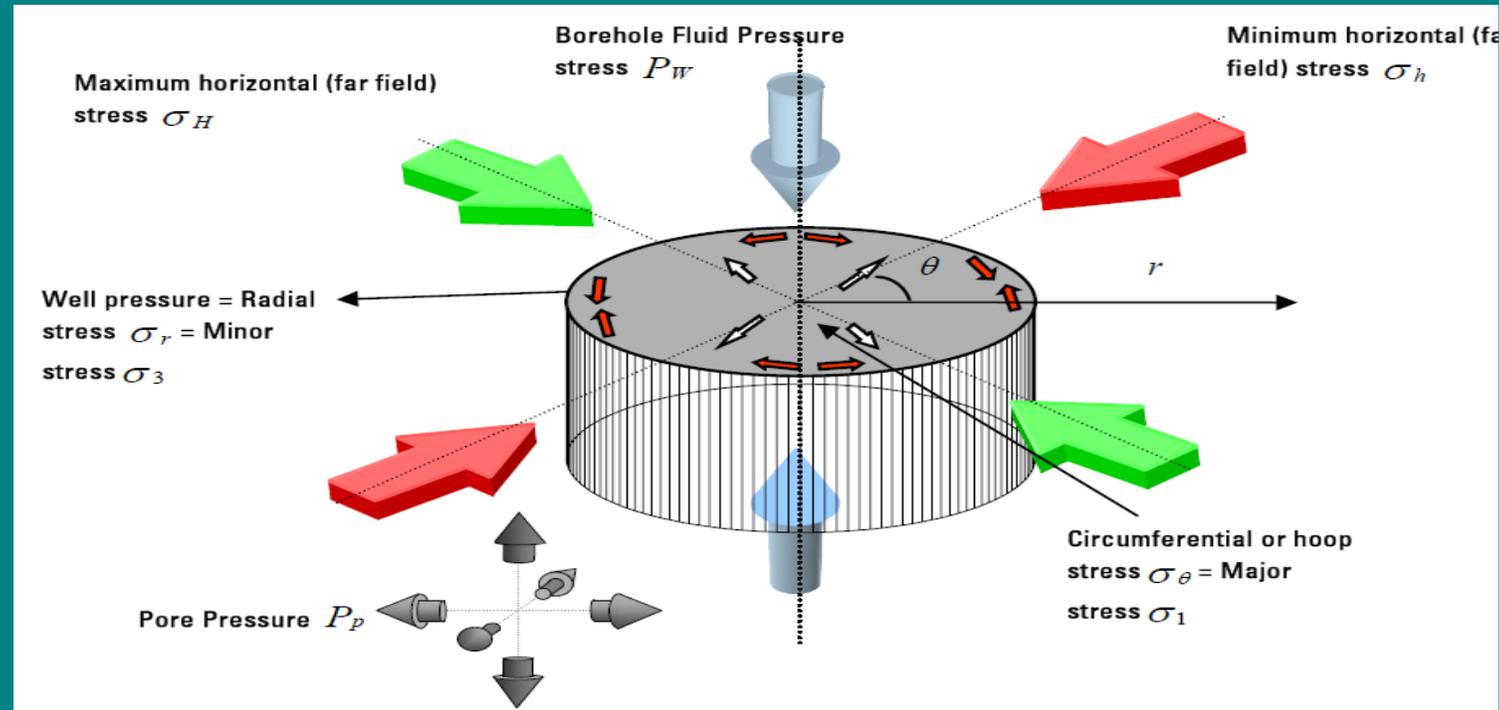
$$\sigma_v = \sum \rho_i g (z_{i+1} - z_i)$$



# Density logs

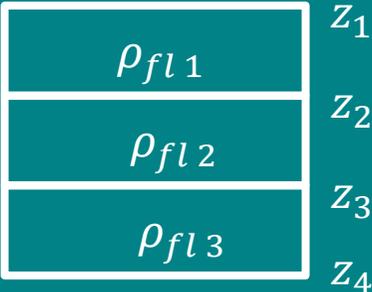


# Stress configuration around borehole – no additional tectonics



# Pore pressure

- Pressure of fluids in porous rock  $P_p = \rho_{fl}gz$
- Acting against lithostatic rock pressure

$$P_p = \sum \rho_{fl i} g (z_{i+1} - z_i)$$


Biot constant, often 1



- Vertical effective stress  $\sigma'_v = \sigma_v - \alpha P_p$

# Vertical stress

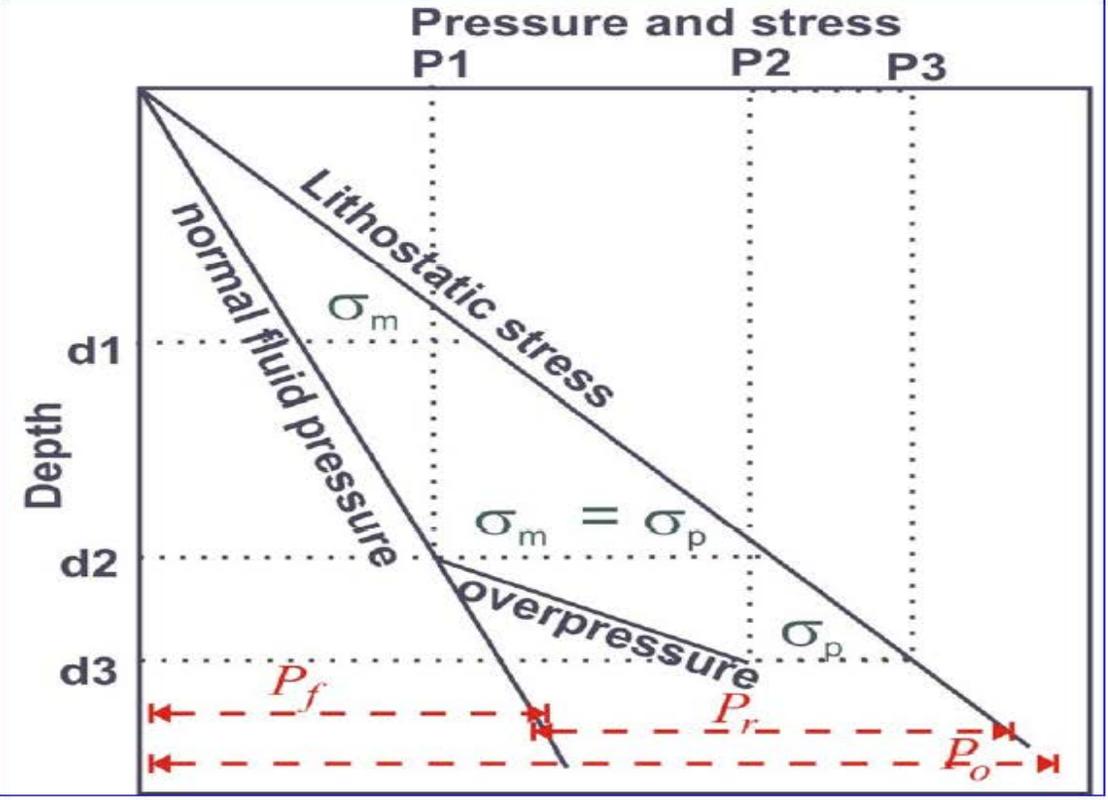
- Average density of upper crustal sediments: 2.3 g/cm<sup>3</sup>
- Volcanics much higher density (2.8 - >3 g/cm<sup>3</sup>)
- Density of water: ~1 g/cm<sup>3</sup>

# Effect of fluid overpressure

In-situ conditions: P,V,T,t: Pressure

Pressure - depth relation for normal compaction and non-equilibrium.

$P_o = P_r + P_f$	$P_o$	Total overburden pressure
	$P_f$	Fluid pressure
$\sigma_p = P_o - P_f$	$P_r$	Lithostatic (grain) pressure
	$\sigma_p$	Effective stress
$\sigma_p = \sigma_m$	$\sigma_m$	Matrix stress or load on the grain framework



# Minimum horizontal stress ( $\sigma_h$ )

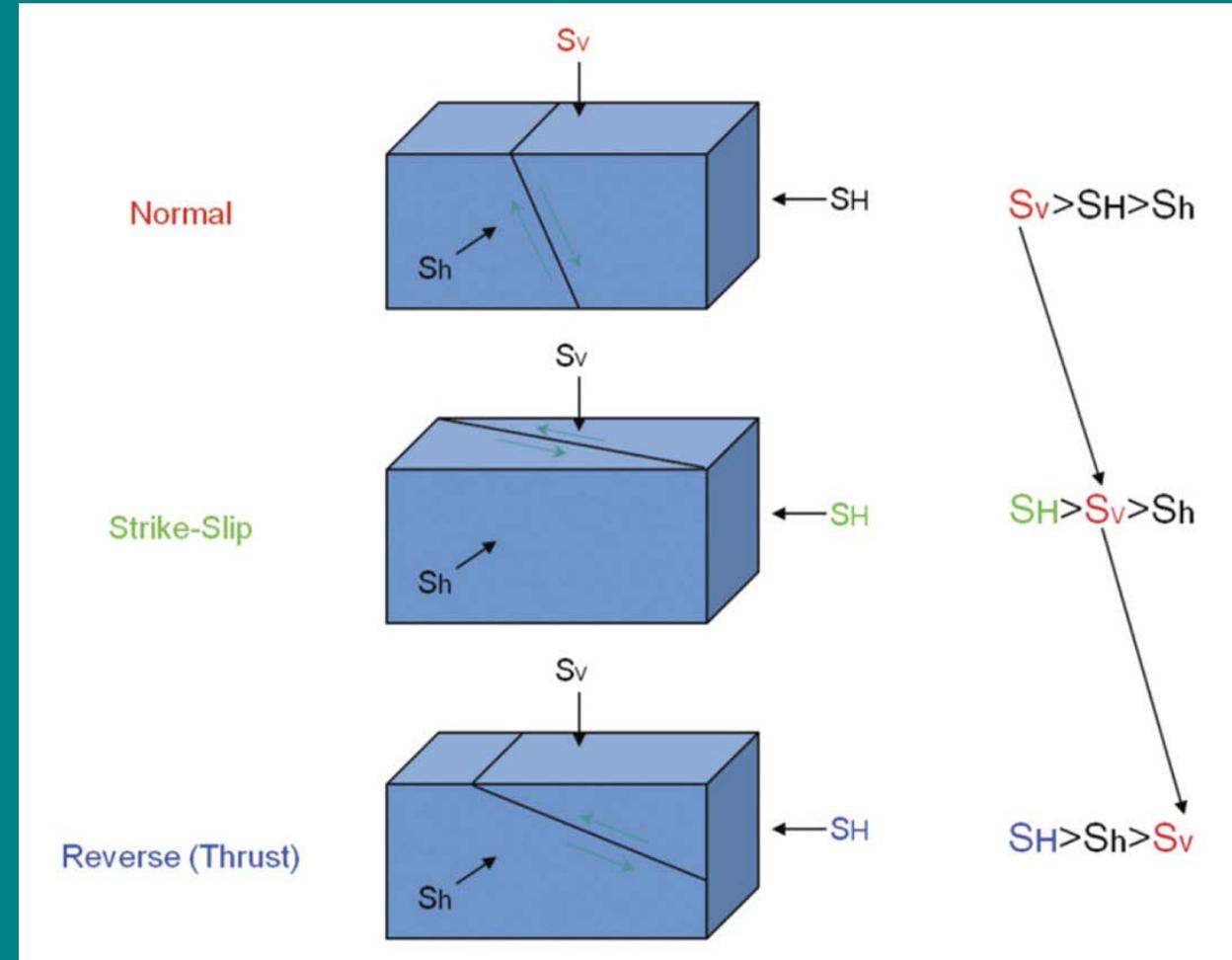
$$\sigma_h = \frac{\nu}{1-\nu} (\sigma_v - \alpha P_p) + \alpha P_p$$

Poisson's ratio

$$\nu = \frac{\nu_p^2 - 2\nu_s^2}{2(\nu_p^2 - \nu_s^2)}$$

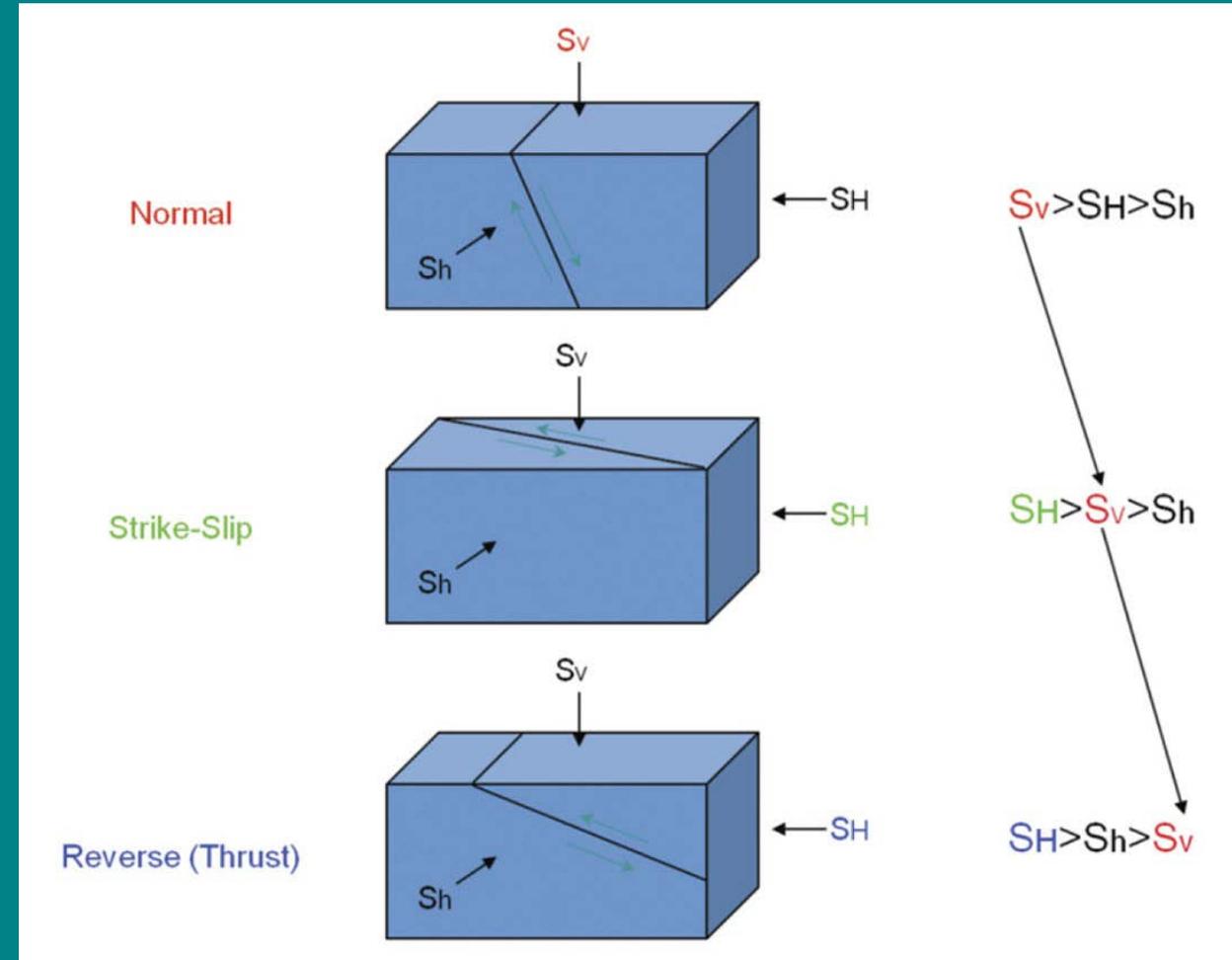
# Maximum horizontal stress ( $\sigma_H$ )

The difference between the maximum and minimum horizontal stress is difficult to determine and depends on the tectonic stresses (are they large or are they small) and the tectonic setting (normal faulting, strike slip, or compression).



# Normal faulting/extension

- In normal faulting scenarios which are also tectonically quiet (e.g. the Netherlands), it is often OK to assume that  $\sigma_h \sim \sigma_H$  and that they are both smaller than  $\sigma_V$ .

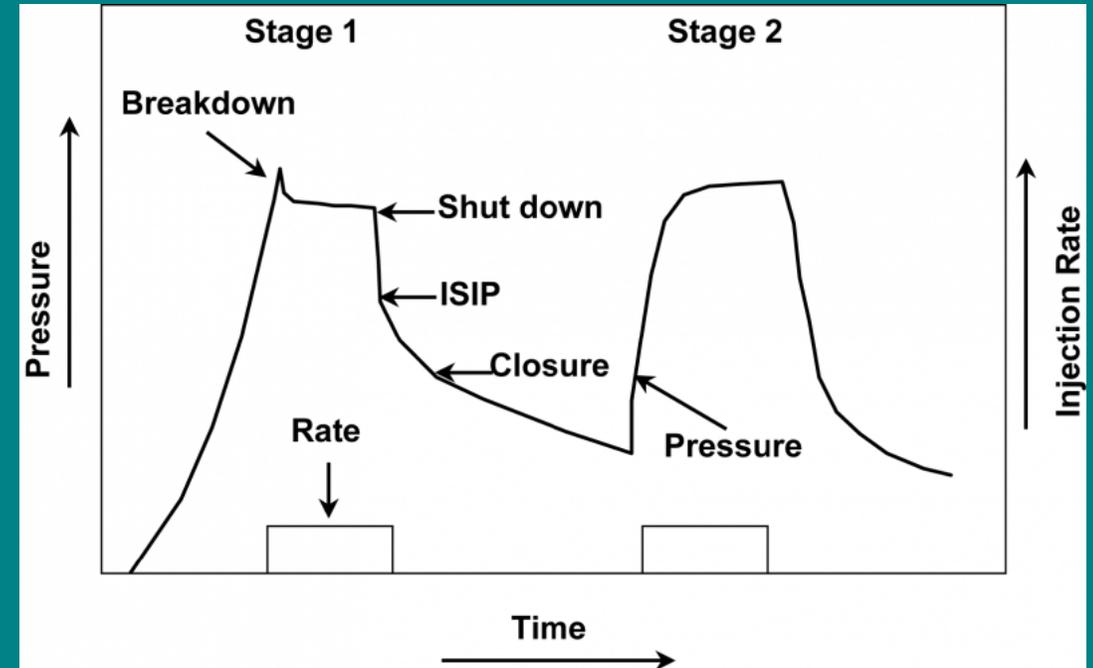
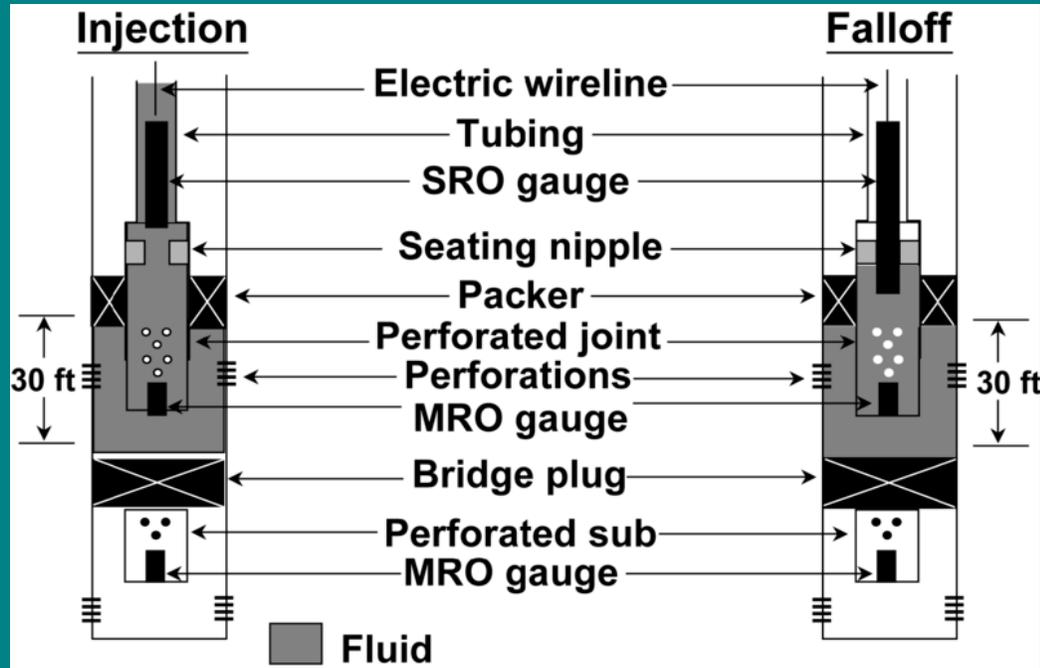


# Important for Indonesia!

- In tectonically active areas, it is even more important to include the effects of tectonic activity in the analyses of the total stresses.
- To measure the tectonic stresses, injection tests are conducted to measure the minimum horizontal stress. The measured stress is then compared with the stress calculated by the poroelastic equation to determine the value of the tectonic stress.

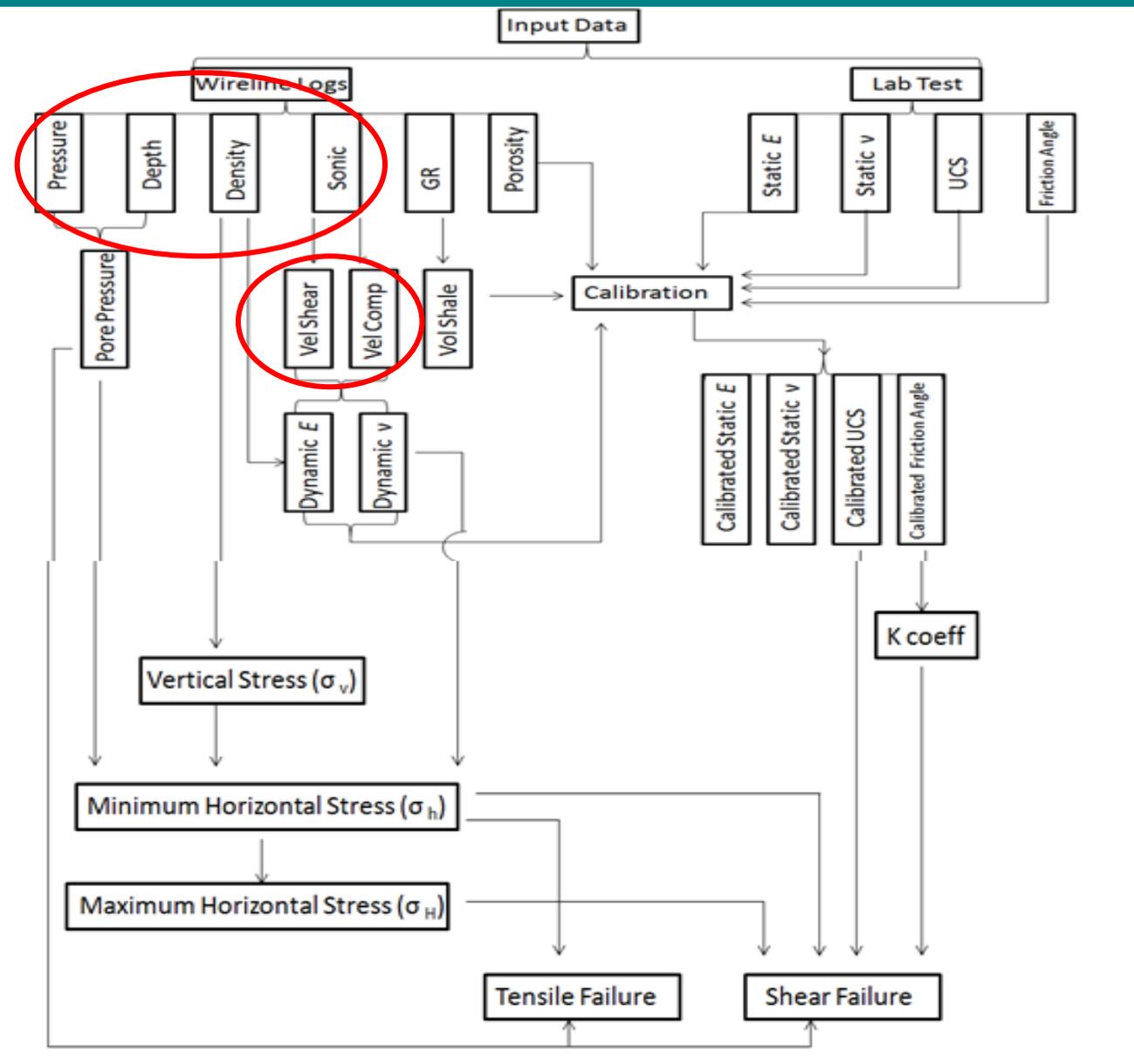
- E.g. 
$$\sigma_h = \frac{\nu}{1-\nu} (\sigma_v - \alpha P_p) + \alpha P_p + \sigma_{tect}$$

# Injection tests



The in-situ stress test is conducted with small volumes of fluid (a few barrels) and injected at a low injection rate (tens of gal/min), normally with straddle packers to minimize wellbore storage effects, into a small number of perforations (1 to 2 ft). The objective is to pump a thin fluid (water or nitrogen) at a rate just sufficient to create a small fracture. Once the fracture is open, the pumps are shut down, and the pressure is recorded and analyzed to determine when the fracture closes. Thus, the term "fracture-closure pressure" is synonymous with minimum in-situ stress and minimum horizontal stress.

[http://petrowiki.org/Fracture\\_mechanics](http://petrowiki.org/Fracture_mechanics)



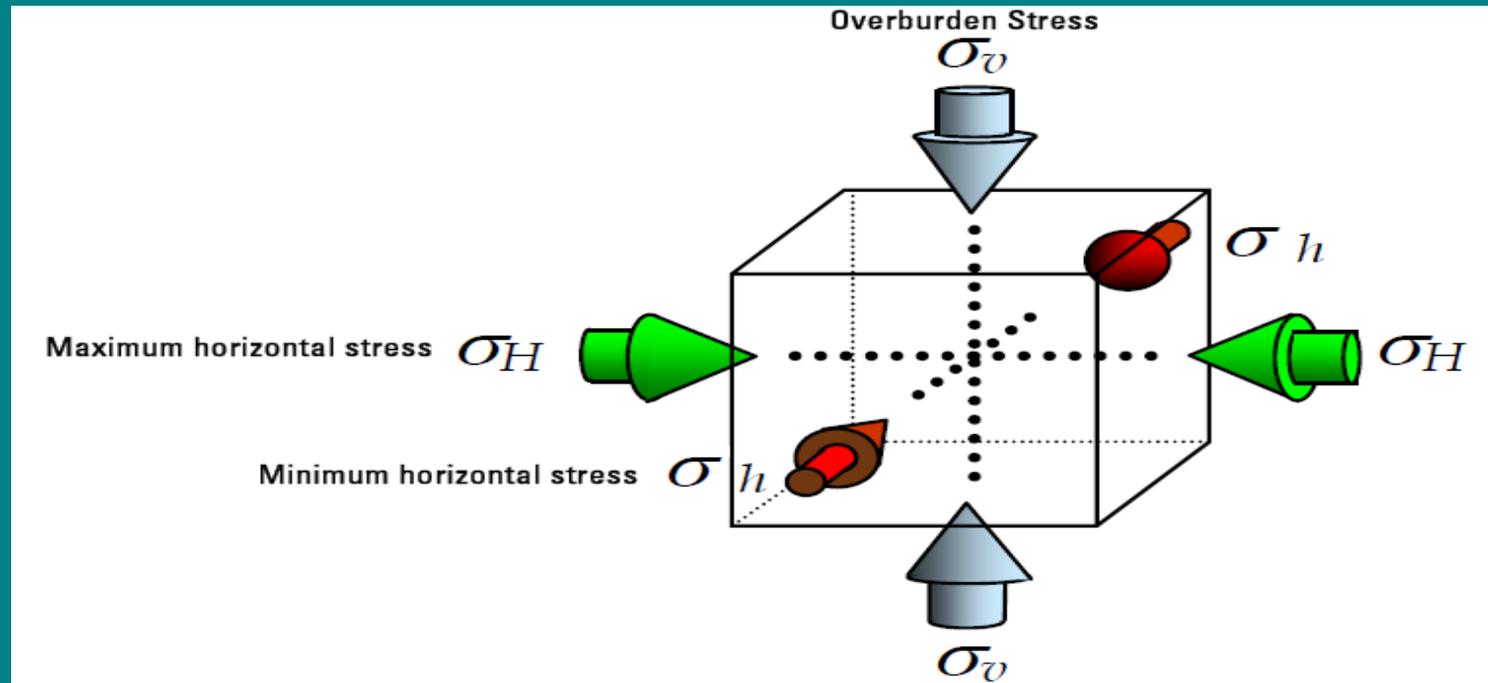
Wireline logs including:

- Depth
- Fluid pressure
- Density
- Compressional velocity
- Shear wave velocity

Enables you to determine stress state and elastic/geomechanical moduli for the reservoir!

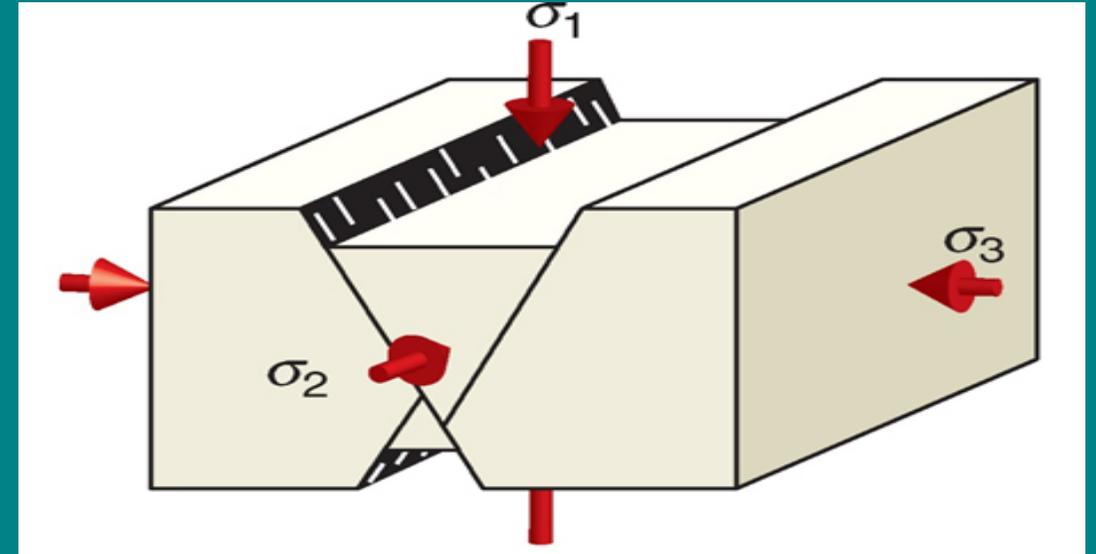
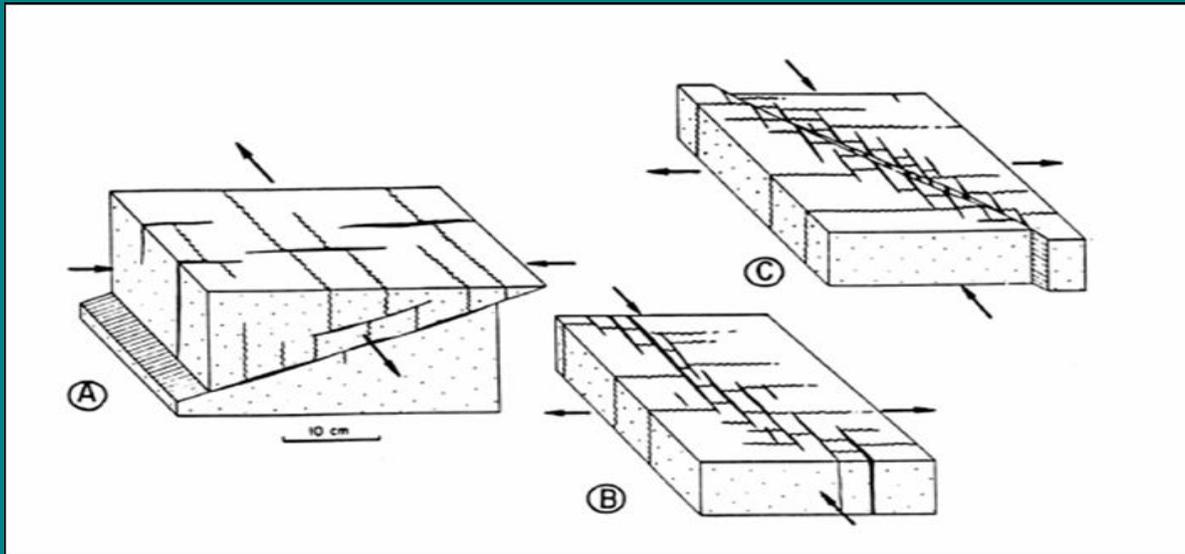
# Stress orientation

- Not from conventional wireline Logs
- Need to come from borehole images

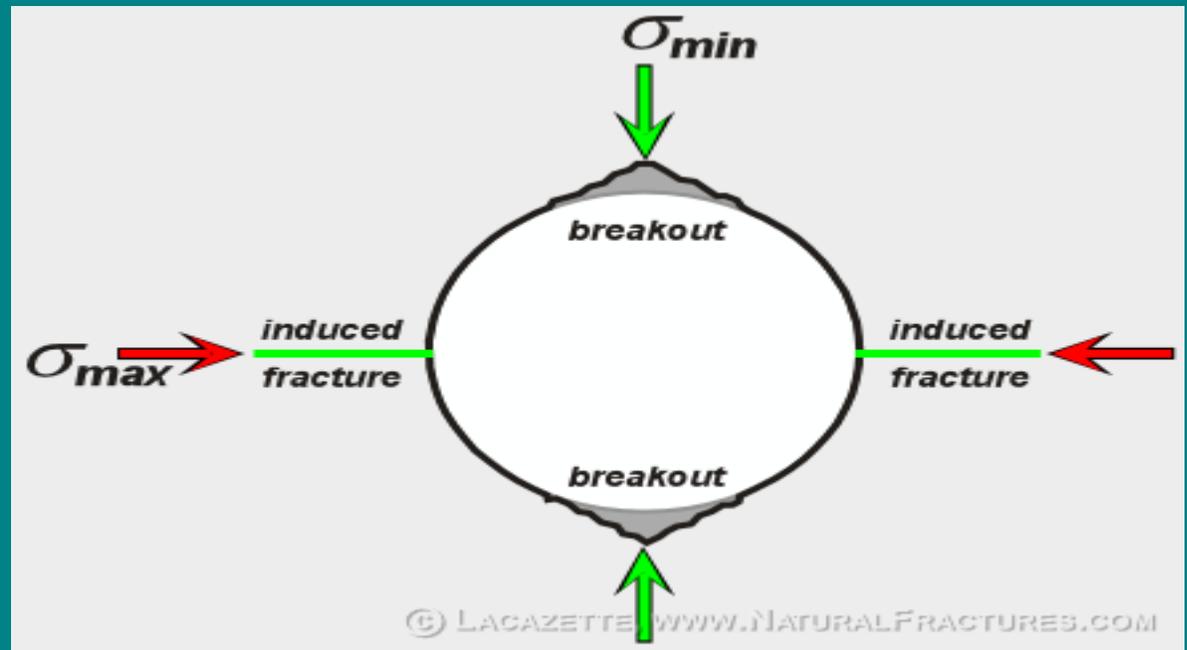


# Extension fracturing

- In the  $\sigma_1 - \sigma_2$  plane, perpendicular to  $\sigma_3$

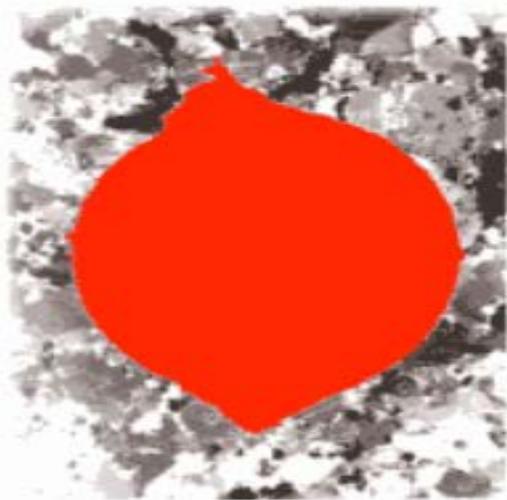


# In borehole



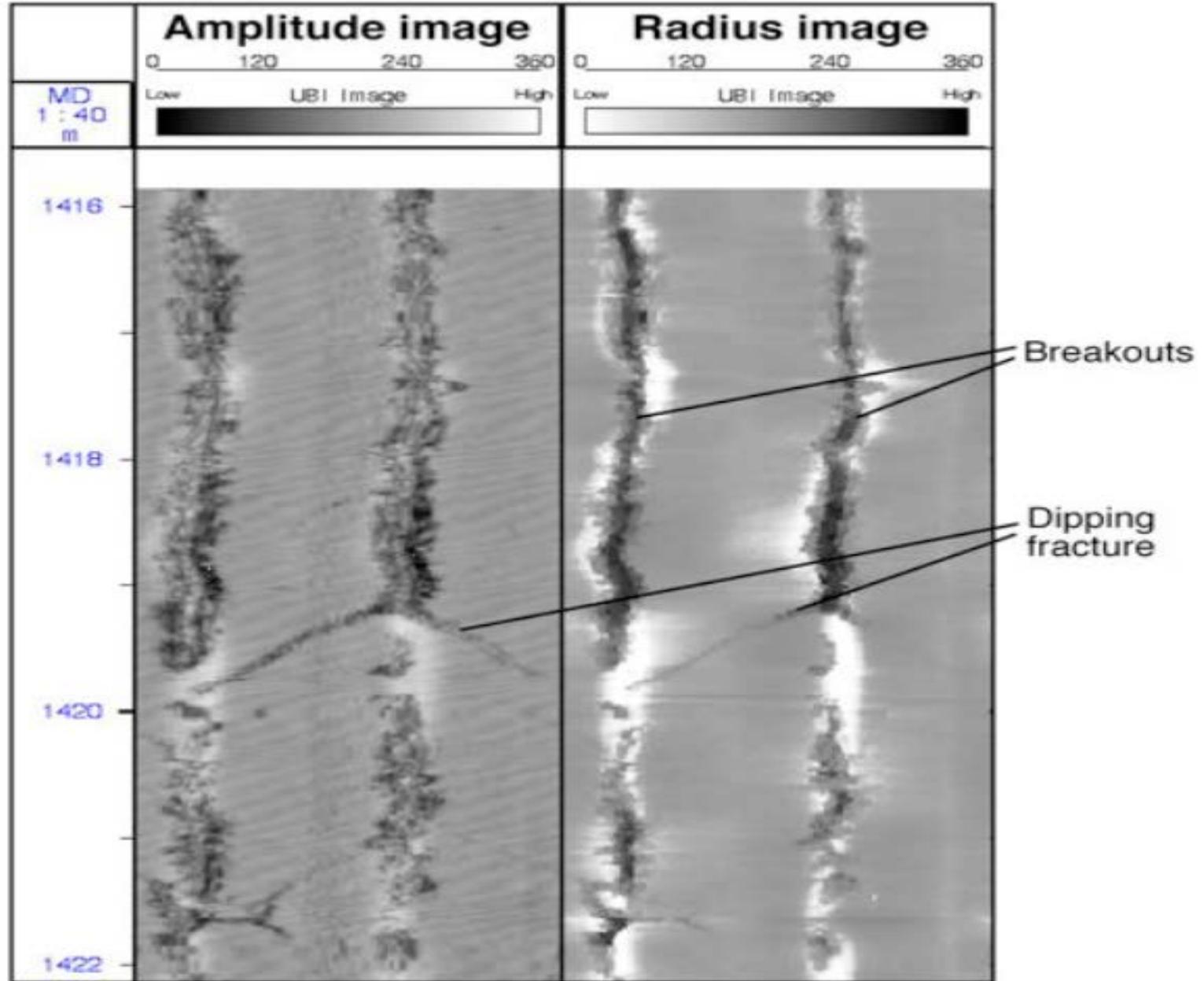
# Borehole Breakouts

Mark the minimum stress direction

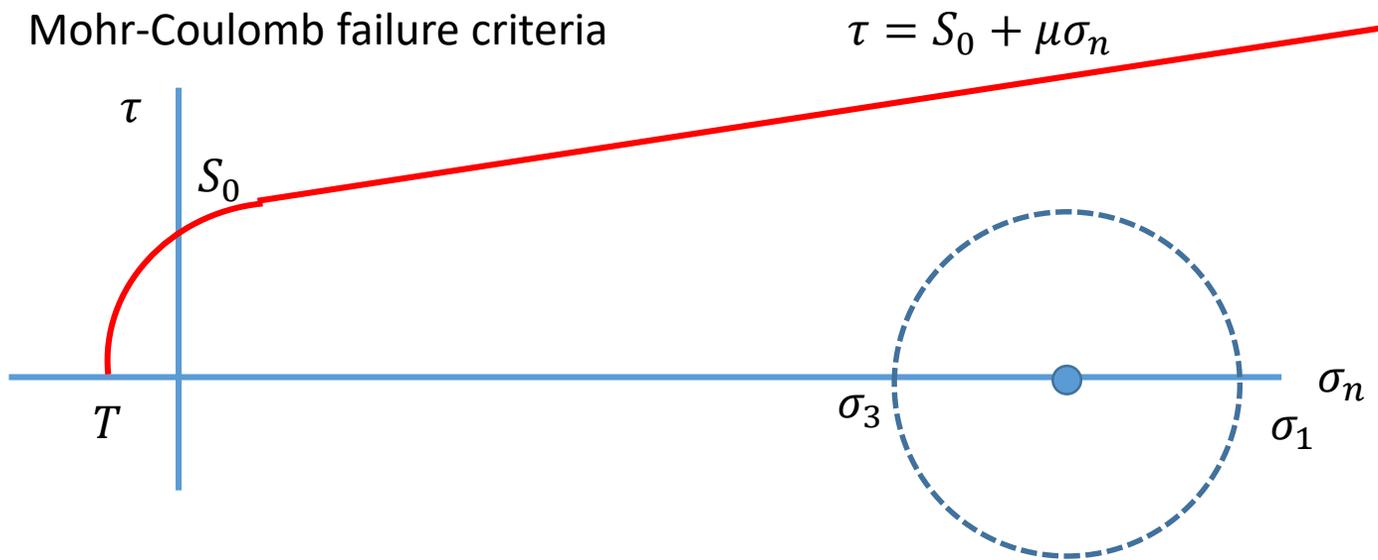


Maximum Horizontal Stress

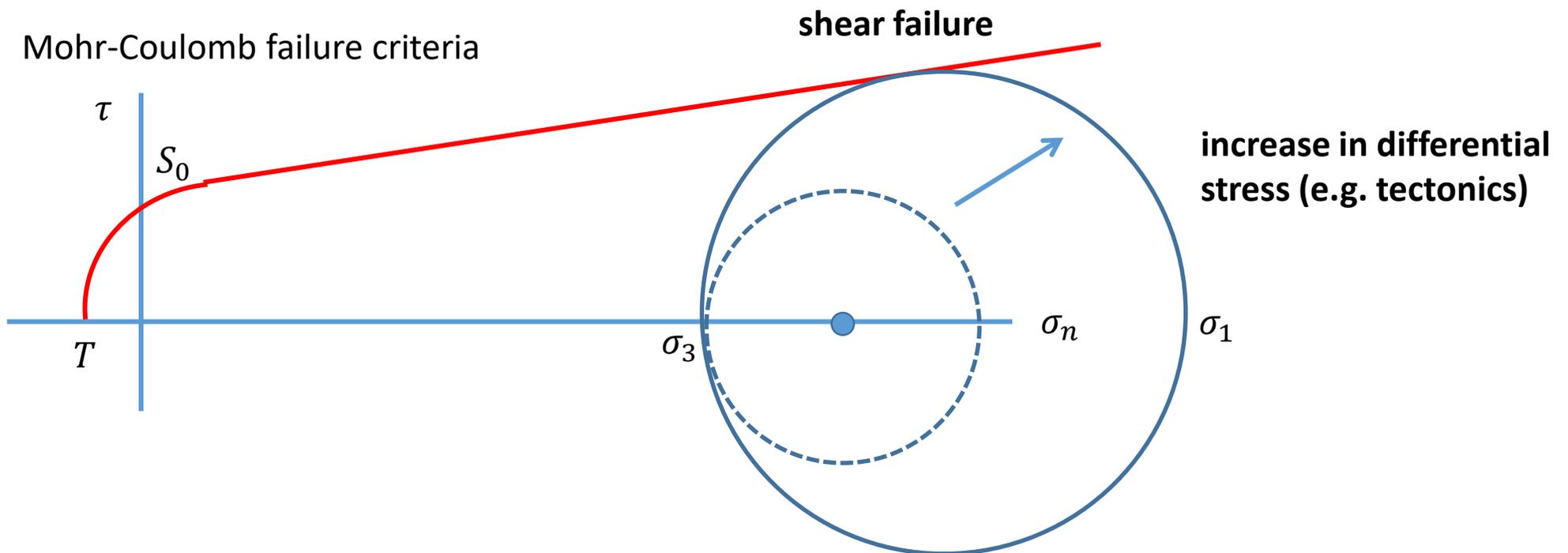
Minimum Horizontal Stress



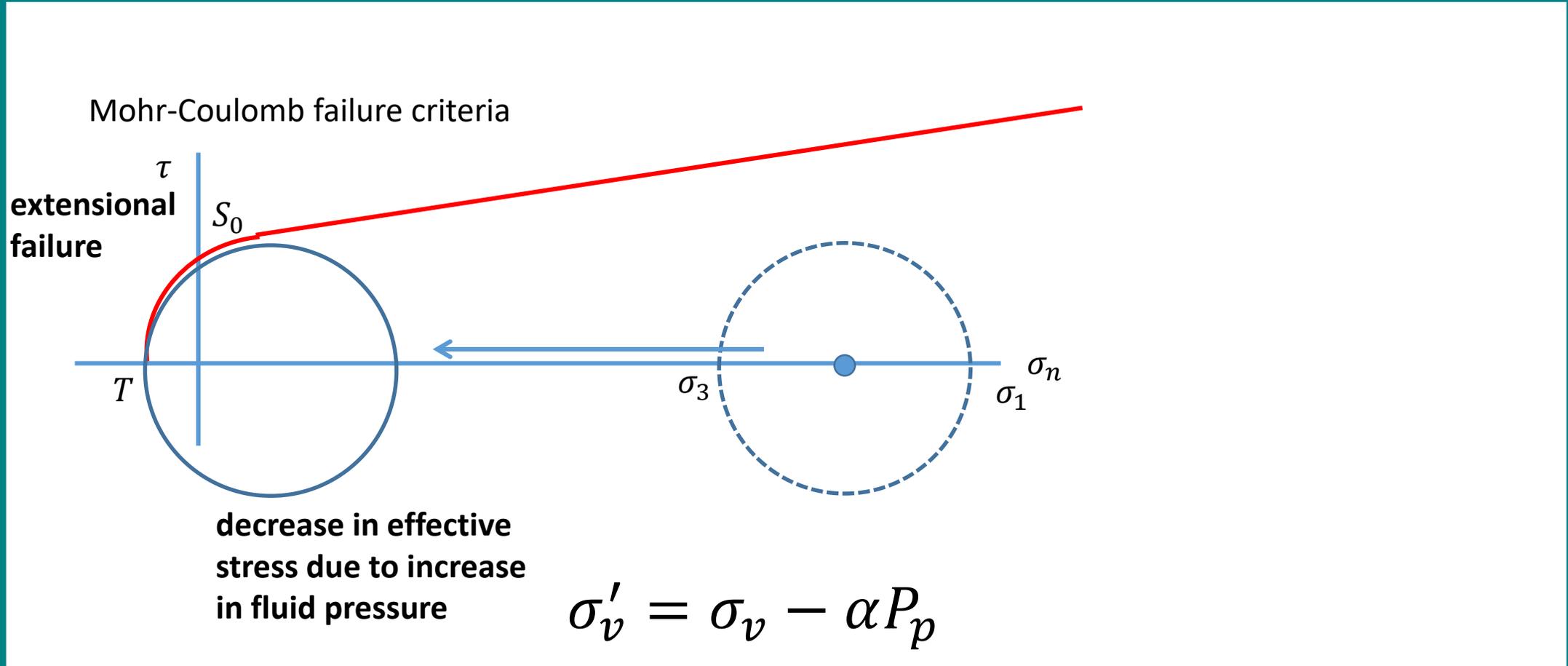
# Mohr-Coulomb failure criterion



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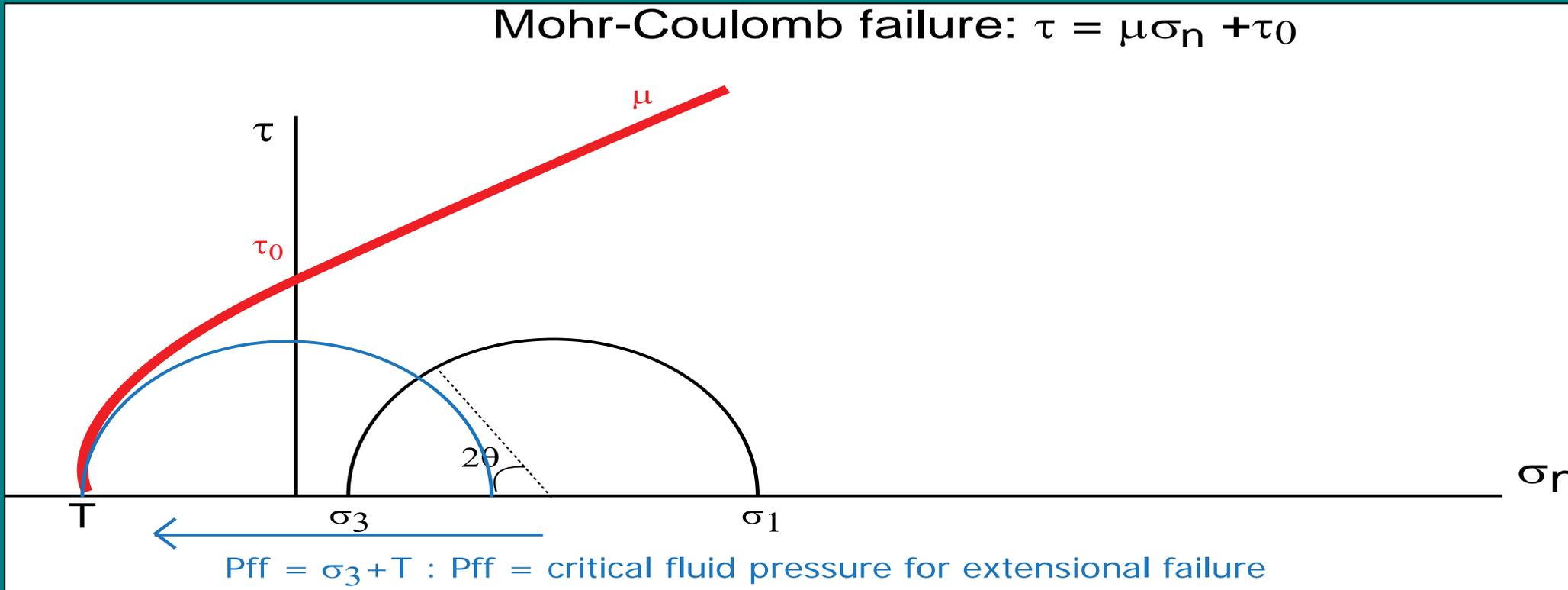


# Hydraulic fracturing

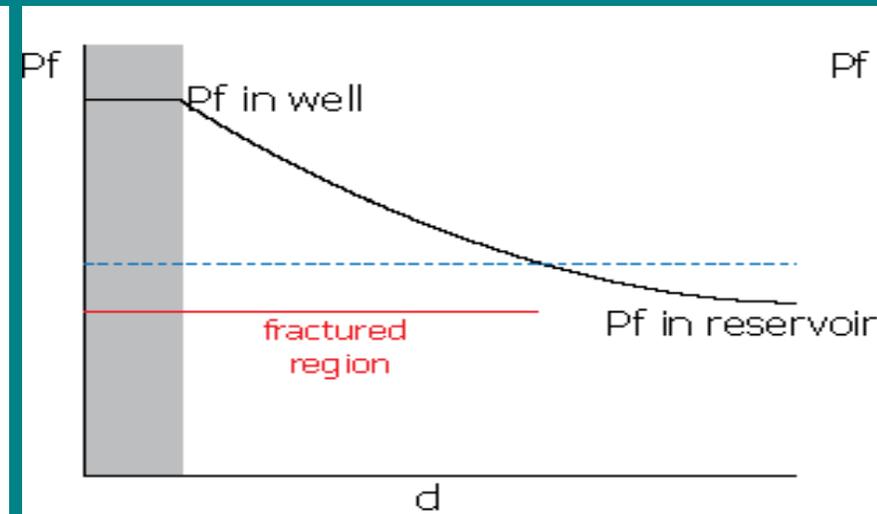
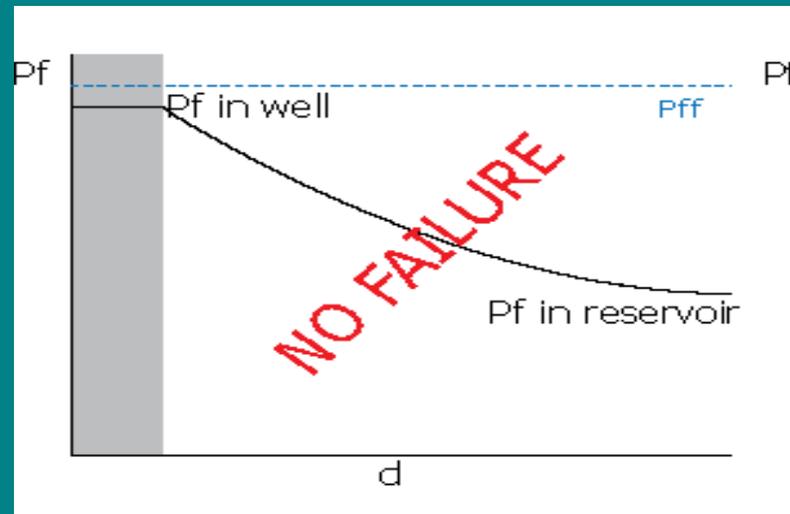
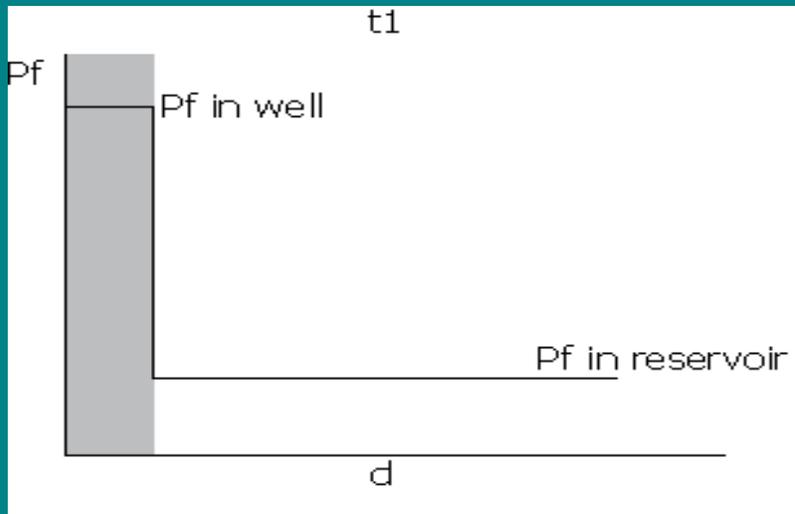
$$P_{ff} = \sigma_h + T$$

- Mohr-coulomb failure criterion

Mohr-Coulomb failure:  $\tau = \mu\sigma_n + \tau_0$



# Fluid pressure in the well



# Excel exercise -1

20 min

Open Excel file named **17052016GEOCAP\_TTT1.04TUD-ReservoirGeomechanicsExerciseStressProfiles\_withoutanswers.xls**

1. Create a vertical stress profile through the Earth for a simplified Earth structure with an average density of  $2.3 \text{ g/cm}^3$  (sandstone with porosity) and a more dense volcanic crust with an average density of  $\sim 3 \text{ g/cm}^3$
2. Create a fluid pressure profile through the same section.
3. If we assume an average Poisson's ratio for the crust of 0.25, what are then the minimum horizontal stresses.
4. How much do we need to increase the borehole fluid pressure to open fractures if the tensile strength of the sandstone is 8 MPa and that of the basalt is 22 MPa?
5. What do those borehole fluid pressures ( $P_{ff}$ ) mean if the intention is to avoid fracturing?

# Excel exercise -2

60 min

A set of logs is available from a well between 3800 and 4400 m. The logs amongst others include density,  $v_p$  and  $v_s$  data and bottomhole pressure.

Open Excel file named **17052016GEOCAP\_TTT1.04TUD-ReservoirGeomechanicsExerciseLoggingdata\_withoutanswers.xls**

1. Create again a vertical stress profile using the log data. For the first 3830m use an average density of  $2.3 \text{ g/cm}^3$ , after that you can use the density of the logs.
2. Calculate the poisson's ratio log using the available data.
3. Calculate the minimum horizontal stress profile for the logged section and again calculate the borehole fluid pressure needed to induce fractures if we assume a tensile strength for the logged section (mainly carbonates) of  $\sim 12 \text{ MPa}$ .
4. Since both  $V_p$ ,  $V_s$  and density are measured you can also create logs for the Young's modulus, Bulk modulus and Shear modulus. How many layers would you assume to be present based on the geomechanical logs and how many based on the density and velocity logs alone. What causes this possible difference?

# Good Books

**Mark Zoback, 2007**

Reservoir Geomechanics Cambridge University Press

**Erling Fjaer et al., 2008**

Petroleum Related Rock Mechanics

**Jaeger et al., 1969**

Fundamentals of Rock Mechanics

**Djebbar Tiab and Erle Donaldson, 2011**

Petrophysics

# Good online geomechanics course

Mark Zoback's online geomechanics course (Stanford, US)

- <https://lagunita.stanford.edu/courses/EarthSciences/ReservGeomech/Spring2016/about>

session 1 -introduction- is also on YouTube:

<https://www.youtube.com/watch?v=vxISqGZ7rAA>

# Thank you!

# Contact Details

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